		...LC circuit with R=50 Ω, ...mH and C=50 μF as a voltage of ... applied to it at t=0 through a ...witch. Evaluate the expression for a current transient. Assume initially relaxed circuit conditions.	L3	CO3	5 M
		OR			
9	a)	Derive an expression for the transient current in series RC circuit with a sinusoidal source using differential equations.	L4	CO3	5 M
	b)	A voltage pulse $v(t) = u(t-2) - u(t-4)$ is applied to a series RL circuit with R = 5 ohms and L = 5 henry. Obtain voltage expression across R and L. Where $u(t)$ is the unit step function.	L3	CO3	5 M
		UNIT-V			
10	a)	Derive the relationship between transmission (ABCD) parameters and open circuit impedance (Z) parameters.	L3	CO5	6 M
	b)	Compute the transmission parameters for the two-port network if the Z parameters for the network are $Z_{11} = 42\Omega$, $Z_{22} = 35\Omega$, $Z_{12} = Z_{21} = 25\Omega$.	L3	CO5	4 M
		OR			
11	a)	Discuss about the image parameters for the symmetrical two port networks.	L3	CO5	6 M
	b)	Derive the expression for image transfer constant (θ) in terms of transmission (ABCD) parameters.	L4	CO5	4 M

Code: 23EC3201

I B.Tech - II Semester – Regular Examinations - JULY 2024

NETWORK ANALYSIS
(ELECTRONICS & COMMUNICATION ENGINEERING)

Duration: 3 hours — Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

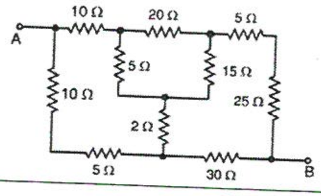
BL – Blooms Level — CO – Course Outcome

PART – A

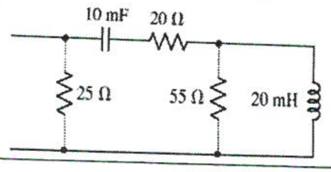
		BL	CO
1.a)	Explain about super node and super mesh.	L2	CO1
1.b)	Draw the phasor diagram of a series RL & RC circuit.	L2	CO1
1.c)	Distinguish between Independent and Dependent Sources.	L2	CO2
1.d)	Discuss the limitations of maximum power transfer theorem.	L2	CO2
1.e)	State the conditions for resonance in a parallel RLC resonant circuit.	L1	CO4
1.f)	Explain the term Coefficient of Coupling in magnetic circuits.	L2	CO4
1.g)	Why Laplace transform method is superior to classical method to solve the differential equations.	L1	CO3
1.h)	Write the second order differential equation that governs the series RLC circuit.	L2	CO3
1.i)	Write the condition for symmetry and reciprocity of a two port network represented in z parameters.	L1	CO5
1.j)	Draw the h-parameter model.	L2	CO5

PART – B

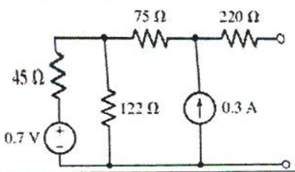
			BL	CO	Max. Marks
		UNIT-I			
2	a)	Derive the equations required to convert a star connected network to delta connected network.	L3	CO1	4 M
	b)	Determine the equivalent resistance between the terminal's 'A' and 'B' of the following network.	L3	CO1	6 M



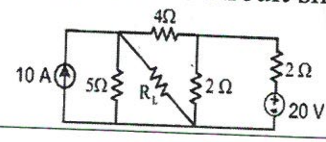
			BL	CO	Max. Marks
		OR			
3	a)	Explain about Mesh analysis and write the steps for mesh analysis.	L2	CO2	4 M
	b)	Determine equivalent impedance seen looking into the open terminals of the network if $\omega = 100$ rad/sec	L3	CO1	6 M



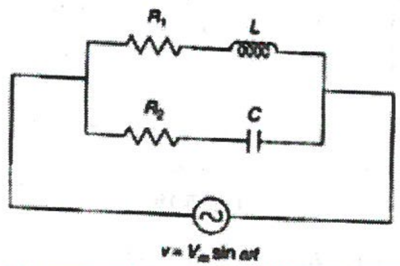
			BL	CO	Max. Marks
		UNIT-II			
4	a)	State and explain the Thevenin's theorem.	L3	CO2	4 M
	b)	Employ Thevenin's theorem to obtain a simple two component equivalent of the circuit shown below.	L3	CO2	6 M



			BL	CO	Max. Marks
		OR			
5	a)	State and explain the super position theorem.	L3	CO2	4 M
	b)	Determine the current through $R_L = 7.5\ \Omega$ resistance using superposition theorem for the circuit shown below.	L3	CO2	6 M



			BL	CO	Max. Marks
		UNIT-III			
6	a)	Draw the series RLC circuit and derive the expression for resonant frequency and bandwidth.	L3	CO4	6 M
	b)	Discuss about the quality factor of a series and parallel resonant circuit.	L4	CO4	4 M
		OR			
7	a)	What are coupled circuits and explain about self and mutual inductance.	L2	CO4	5 M
	b)	Derive the expression for the resonant frequency of the given circuit.	L3	CO4	5 M



			BL	CO	Max. Marks
		UNIT-IV			
8	a)	Derive the unit step current response of series RLC circuit.	L4	CO3	5 M

Code: 23EC3251

I B.Tech – II Semester - Regular Examinations - JULY 2024

NETWORK ANALYEIS

(ELECTRONICS & COMMUNICATION ENGINEERING)

Scheme of Valuation

PART-A

1 a) super node – 1M ; super Mesh – 1M

b) RL diagram – 1M ; RC diagram – 1M

c) Representation of each – 1M + 1M

d) Limitations – 2M

e) condition – 2M

f) explanation – 2M

g) Advantages of laplace – 2M

h) Equation – 2M

i) symmetry condition – 1M ; Reciprocity – 1M

j) Model – 2M

PART-B

2 a) Formulae – 2M ; Diagram – 1M ; Derivation – 2M

b) substitution & simplification – 5M

Answer – 1M

3 a) Explanation – 4M

b) Finding X_L, X_C – 2M ;

simplification – 3M ;

Answer – 1M

) statement - 2M
Explanation - 2M

b) substitution & simplification - 5M
Answer - 1M

5a) Statement - 2M
Explanation - 2M

b) substitution & simplification - 5M
Answer - 1M

6 a) frequency - 3M ; Bandwidth - 3M

b) Quality factor of series ckt - 2M
Parallel ckt - 2M

7 a) Coupled circuits - 1M
Self inductance - 2M
Mutual inductance - 2M

b) Derivation - 5M

8 a) Derivation - 5M

b) substitution & simplification - 4M ; Answer - 1M

9 a) Derivation - 5M

b) substitution & simplification - 4M; Answer - 1M

10 a) Derivation - 6M

b) substitution & simplification - 4M

11 a) Explanation - 6M

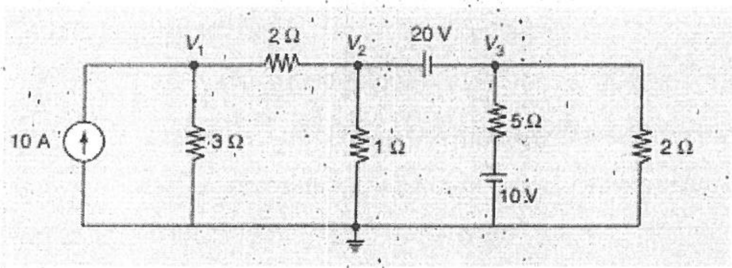
b) Derivation - 4M

PART — A

1.a) Explain about super node and super mesh

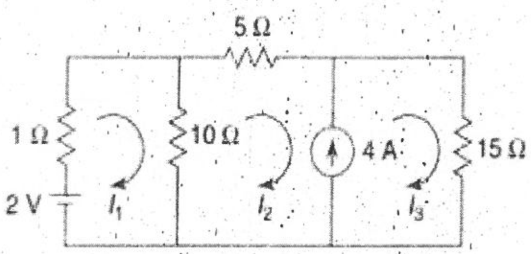
super node: Nodes that are connected to each other by voltage sources, but not to the reference node by a path of voltage sources, form a super node.

Example: In this circuit node 2 and 3 form a super node.



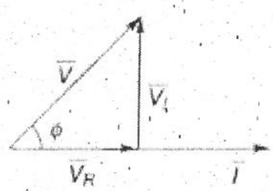
super mesh: Meshes that share a current source with other meshes, none of which contains a current source in the outer loop, form a super mesh.

Example: In this circuit mesh 2 and 3 form a super mesh.

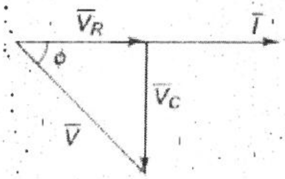


1.b) Draw the phasor diagram of series RL and RC circuit:

Phasor diagram of series RL:

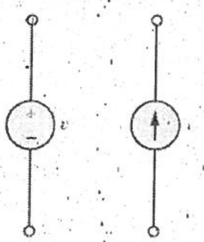


Phasor diagram of series RC:

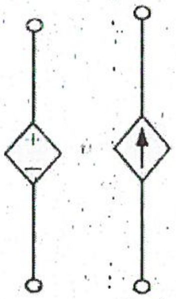


1.c) Distingwish between Independent and Dependent sources:

Independent Sources: Output characteristics of independent sources are not dependent on any network variable such as a current or voltage.



Dependent Sources: If the voltage or current of a source depends upon some other voltage or current, it is called as dependent or controlled source.



1.d) Discuss the limitations of maximum power transfer theorem.

Limitations of maximum power transfer theorem:

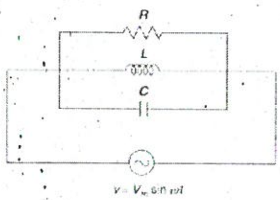
1. It cannot be used in non linear and unilateral networks.
2. The maximum efficiency is 50% and not applicable for power systems.

1.e) State the condition for resonance in parallel RLC circuit.

The condition for resonance is "The imaginary part of the impedance of the circuit must be equal to zero".

Example:

For this circuit resonance condition is



$$\frac{1}{X_L} - \frac{1}{X_C} = 0$$

i.e $\therefore$ $X_L = X_C$

Note: Consider any other parallel circuit also.

1.f) Explain the term Coefficient of Coupling in magnetic circuits:

The coefficient of coupling (k) between coils is defined as "fraction of magnetic flux produced by the current in one coil that links the other".

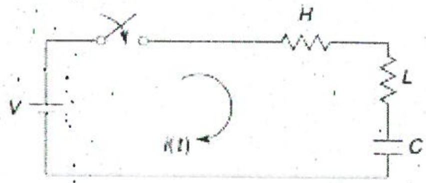
1. g) Why Laplace transform method is superior to classical method to solve the differential equations:

The Laplace transform method has the following **advantages:**

1. Solution of differential equations is a systematic procedure.
2. Initial conditions are automatically incorporated.
3. It gives the complete solution in one step.

1.h) Write the second order differential equation that governs the series RLC circuit.

$$V - Ri - L\frac{di}{dt} - \frac{1}{C}\int_0^t i\,dt = 0$$



Differentiating the above equation,

$$0 - R\frac{di}{dt} - L\frac{d^2i}{dt^2} - \frac{1}{C}i = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$$

1.i) Write the condition for symmetry and reciprocity of a two port network represented in Z parameters.

The condition for symmetry is $Z_{11} = Z_{22}$

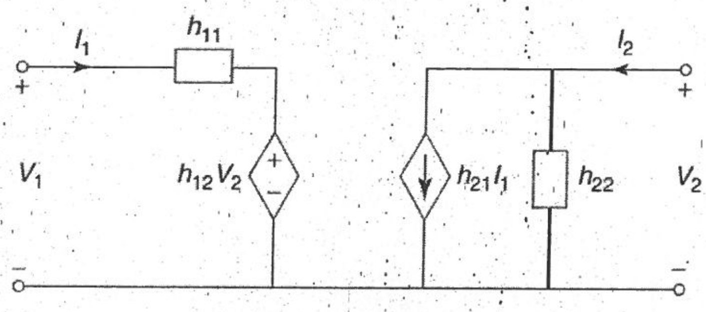
The condition for reciprocity is $Z_{21} = Z_{12}$;

1.j) Draw the h-parameter model.

$$(V_1, I_2) = f(I_1, V_2)$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$

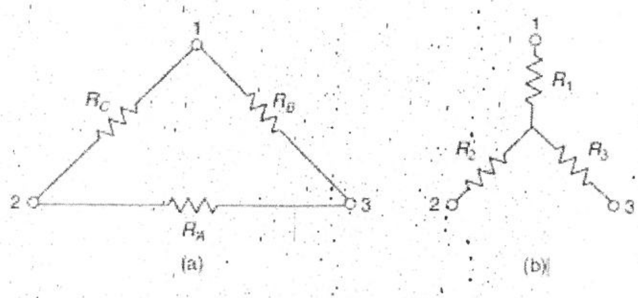
$$I_2 = h_{21} I_1 + h_{22} V_2$$



PART-B

2. a) Derive the equations required to convert a star connected network to delta connected network.

Figure (a) shows three resistors R_A, R_B and R_C connected in delta. Figure (b) shows three resistors R_1, R_2 and R_3 connected in star.



(a) Delta networks (b) Star Network

Referring to delta network shown in Fig. (a), the resistance between terminals 1 and 2

$= R_C \parallel (R_A + R_B) = \dfrac{R_C(R_A + R_B)}{R_A + R_B + R_C}$

Referring to the star network shown in Fig. 1.70 (b), the resistance between terminals 1 and 2 = $R_1 + R_2$.
Since the two networks are electrically equivalent,

$$R_1 + R_2 = \frac{R_C(R_A + R_B)}{R_A + R_B + R_C} \quad \text{...(1.1)}$$

Similarly,

$$R_2 + R_3 = \frac{R_A(R_B + R_C)}{R_A + R_B + R_C} \quad \text{...(1.2)}$$

and

$$R_3 + R_1 = \frac{R_B(R_A + R_C)}{R_A + R_B + R_C} \quad \text{...(1.3)}$$

Subtracting Eq. (1.2) from Eq. (1.1),

$$R_1 - R_3 = \frac{R_B R_C - R_A R_B}{R_A + R_B + R_C} \quad \text{...(1.4)}$$

Adding Eq. (1.4) and Eq. (1.3),

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$

Similarly,

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

Thus, star resistor connected to a terminal is equal to the product of the two delta resistors connected to the same terminal divided by the sum of the delta resistors.

Star to Delta Transformation

Multiplying the above equations,

$$R_1 R_2 = \frac{R_A R_B R_C^2}{(R_A + R_B + R_C)^2} \quad \text{...(1.5)}$$

$$R_2 R_3 = \frac{R_A^2 R_B R_C}{(R_A + R_B + R_C)^2} \quad \text{...(1.6)}$$

$$R_3 R_1 = \frac{R_A R_B^2 R_C}{(R_A + R_B + R_C)^2} \quad \text{...(1.7)}$$

Adding Eqs (1.5), (1.6) and (1.7),

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_A R_B R_C (R_A + R_B + R_C)}{(R_A + R_B + R_C)^2} = \frac{R_A R_B R_C}{R_A + R_B + R_C}$$

$$= R_A R_1 = R_B R_2 = R_C R_3$$

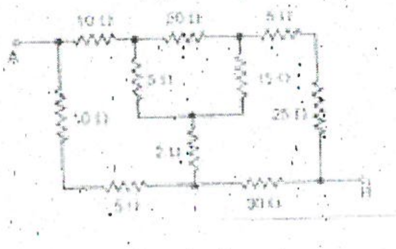
Hence,

$$R_A = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_B = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} = R_3 + R_1 + \frac{R_3 R_1}{R_2}$$

$$R_C = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

2.b) Determine the equivalent resistance between the terminals 'A' and 'B' of the following.

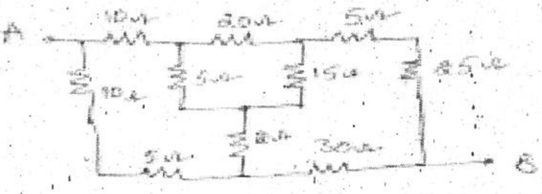


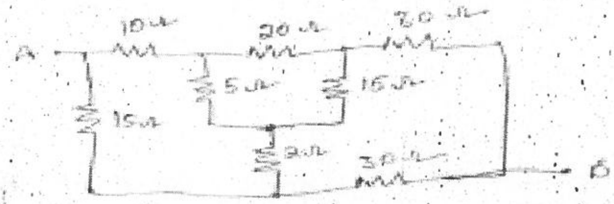
2.b

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$

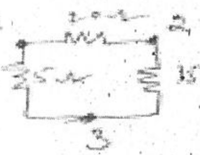
$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

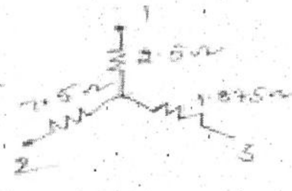




Converting



into star



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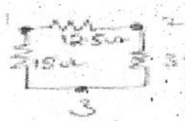
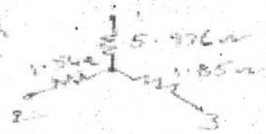
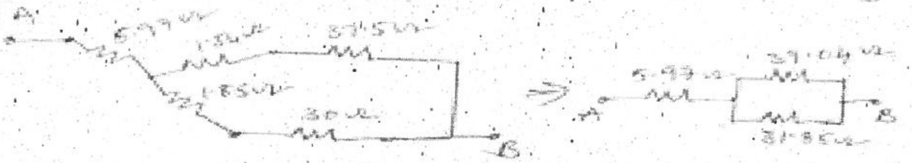


into star





$$R_{AB} = 5.97\Omega + 39.04\Omega \parallel 31.85\Omega$$

$$= 5.97\Omega + \frac{39.04 \times 31.85}{39.04 + 31.85} = \underline{23.51\Omega}$$

3.a) Explain about mesh analysis and write the steps for mesh analysis.

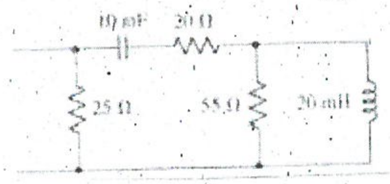
MESH ANALYSIS:

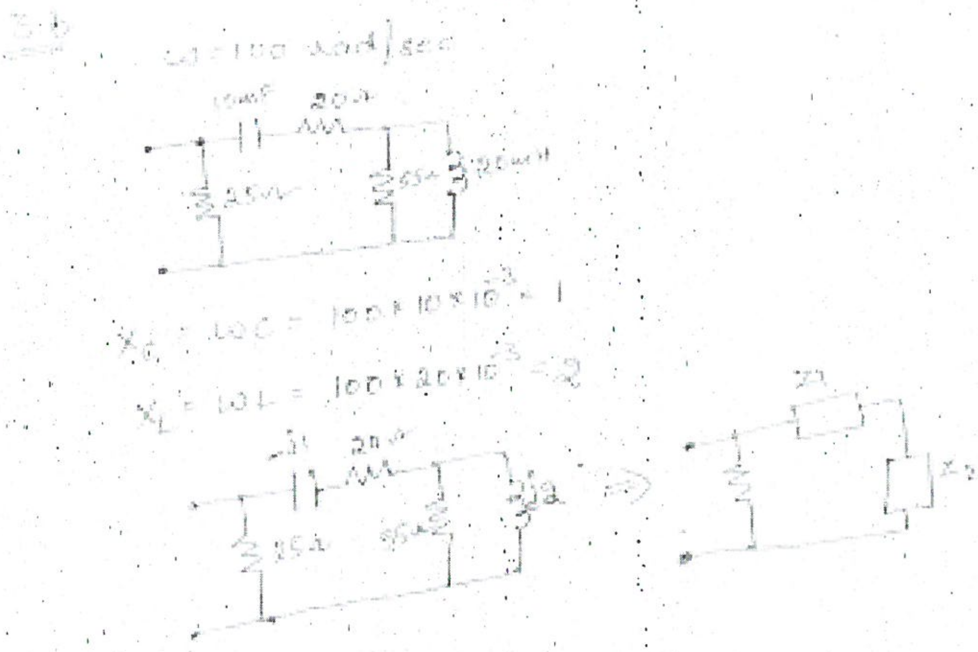
- Mesh is basic important technique used in finding **solutions for a network.**
- If a network has a large number of voltage sources, it is useful to use mesh analysis. Basically, this analysis consists of writing mesh equations by Kirchhoff's voltage law in terms of unknown mesh currents.
- In this method, the currents in different meshes are assigned continuous paths so that they do not split at a junction into branch currents.
- Mesh analysis is applicable only for **planar networks.**

Steps to be followed in Mesh Analysis

1. Identify the mesh, assign a direction to it and assign an unknown current in each mesh.
2. Assign the polarities for voltage across the branches.
3. Apply KVL around the mesh and use Ohm's law to express the branch voltages in terms of unknown mesh currents and the resistance.
4. Solve the simultaneous equations for unknown mesh currents.

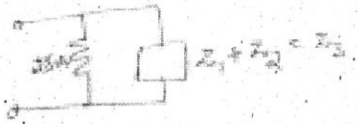
3.b) Determine equivalent impedance seen looking into the open terminals of the network if w=100rad/sec.





$Z_1 = 20 - j1$

$Z_2 = (55) \| (j2) = \frac{55 \times 2j}{55 + 2j} = \frac{110j}{55 + 2j}$



$\therefore Z_3 = Z_1 + Z_2$

$= 20 - j1 + \frac{110j}{55 + 2j}$

$= \frac{1100 - 55j + 40j + 2 + 110j}{55 + 2j}$

$Z_3 = \frac{1102 + 95j}{55 + 2j}$

The impedance looking into the terminals of the network is

$Z = 25 \| Z_3 = \frac{25 \times Z_3}{25 + Z_3}$

$$Z = 11.14 \angle 1.65^\circ$$

4.a) State and explain the Thevenin's theorem.

Thevenin's theorem states that "Any two terminal linear network having a number of voltage current sources and resistances can be replaced by a simple equivalent circuit consisting of a single voltage source in series with a resistance."

where the value of the voltage source is equal to the open-circuit voltage across the two terminals of the network, and resistance is equal to the equivalent resistance measured between the terminals with all the energy sources are replaced by their internal resistances".

- According to Thevenin's theorem, an equivalent circuit can be found to replace the circuit in Fig.

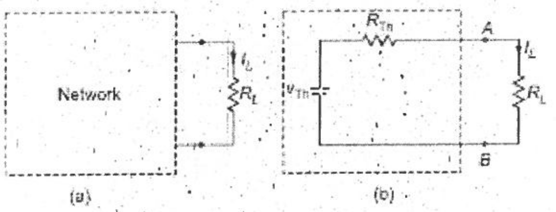


Fig. : Thevenin's Theorem

Steps to be followed in Thevenin's Theorem

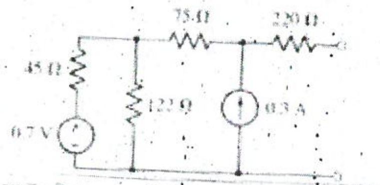
1. Remove the load resistance R_L.
2. Find the open circuit voltage V_{Th} across points A and B.

3. Find the resistance R_{Th} as seen from points A and B with the voltage sources and current sources replaced by internal resistances.
4. Replace the network by a voltage source V_{Th} in series with resistance R_{Th}.
5. Find the current through R_L using Ohm's law.

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

- The above method of determining the load current through a given load resistance can be explained with the help of the following circuit.

4.b) Employ Thevenin's theorem to obtain a simple two component equivalent of the circuit shown below.



4 b :

To find V_{Th} consider the mesh analysis

$i_3 = 0$ as the terminals are open circuited

KVL for mesh 1

$0.7 = 45 i_1 + 122(i_1 - i_2)$

$167 i_1 - 122 i_2 = 0.7$ —— (1)

Mesh 2 & 3 there is supermesh

$i_3 - i_2 = 0.3$

$-i_2 = 0.3 \Rightarrow i_2 = -0.3 A$ —— (2)

Writing KVL to the outer loop of supermesh

$122(i_2 - i_1) + 75 i_2 + V_{Th} = 0$ —— (3)

From eq's (1), (2) & (3) $V_{Th} = 32.875 V$

To find R_{Th}, making all sources as zero

$R_{Th} = (45 \| 122) + 75 + 220$

$= 327.87 \Omega$

5.a) State and explain the super position theorem.

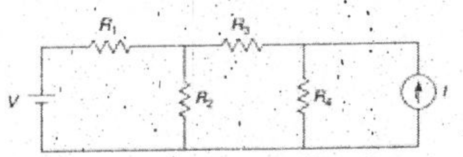
The superposition theorem states that **in any linear network containing two or more sources, the response in any element is equal to the algebraic sum of the responses caused by individual sources acting alone, while the other sources are non-operative;**

That is, while considering the effect of individual sources, other ideal voltage sources and ideal current sources in the network are replaced by short circuit and open circuit across their terminals.

This theorem is valid only for linear systems. This theorem can be better understood with a numerical example.

Consider the network shown in Fig. 3.1.

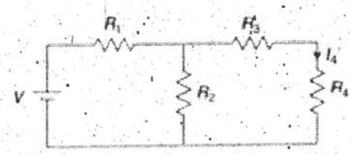
Suppose we have to find current I_4 through resistor R_4



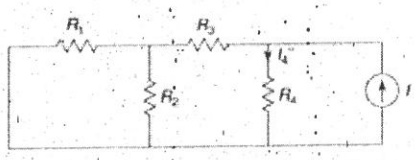
Network to illustrate superposition theorem

- The current flowing through resistor R_4 due to constant voltage source V is found to be say I'_4 A (with proper direction), representing constant current source with infinite resistance, i.e., open circuit.
- The current flowing through resistor R_4 due to constant current source I is found to be say I_4'' (with proper direction), representing the constant voltage source with zero resistance or short circuit.
- The resultant current I_4 through resistor R_4 is found by superposition theorem.

$$I_4 = I_4' + I_4''$$



When voltage source, V is acting alone



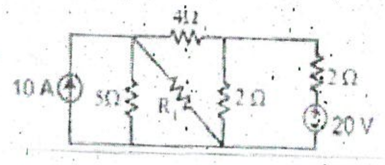
When current source I is acting alone

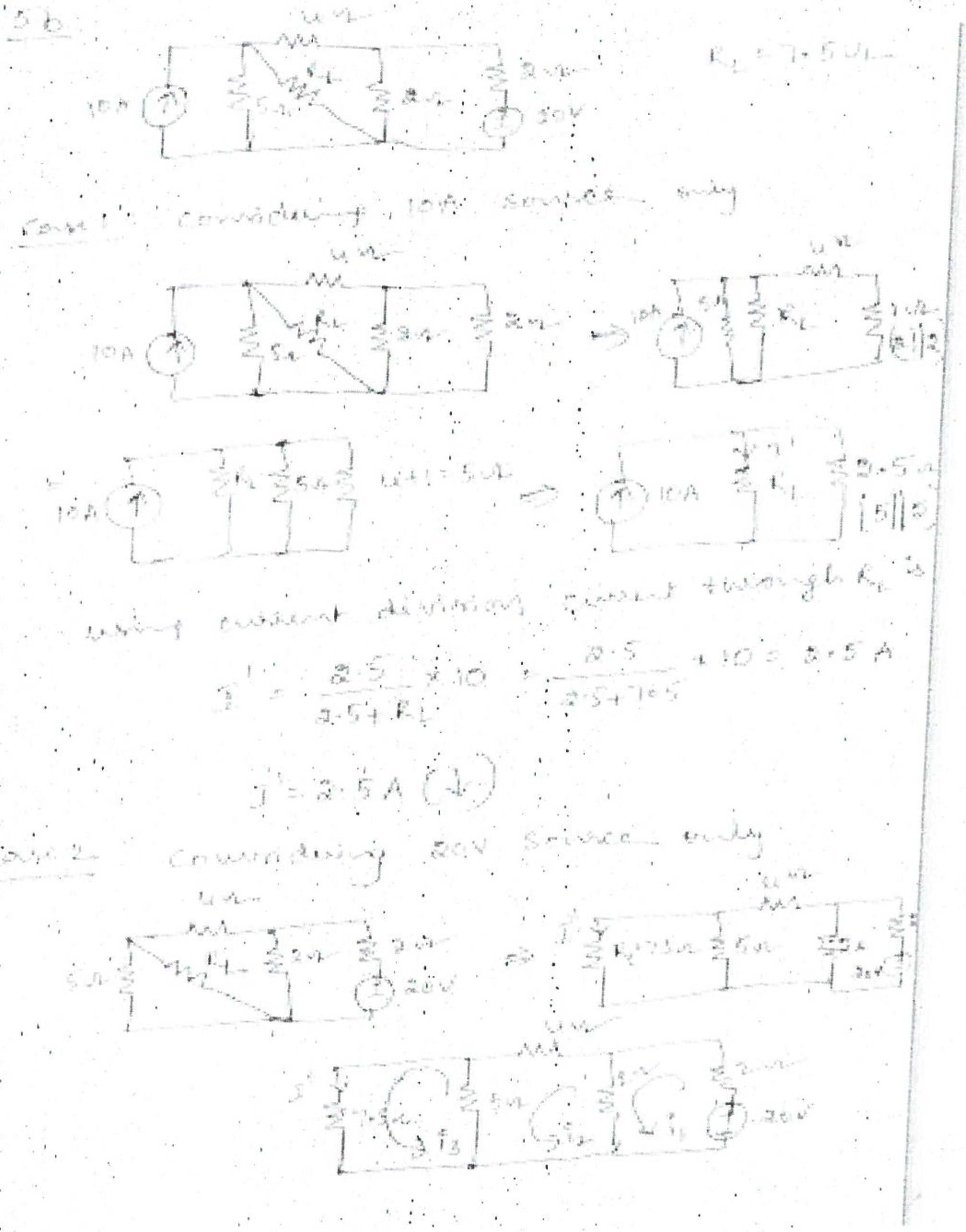
Steps to be followed in Superposition Theorem:

1. Find the current through the resistance when only one independent source is acting, replacing all other independent sources by respective internal resistances.
2. Find the current through the resistance for each of the independent sources.

3. Find the resultant current through the resistance by the superposition theorem considering magnitude and direction of each current.

5.b) Determine the current through $R_L = 7.5\Omega$ resistance using super position theorem for the circuit shown below.





writing mesh equations

Mesh 1 $20 = 2i_1 + 2i_1 - 2i_2 \Rightarrow 2i_1 - i_2 = 10$ — (1)

Mesh 2 $4i_2 + 5(i_2 - i_3) + 2(i_2 - i_1) = 0$

$-2i_1 + 11i_2 - 5i_3 = 0$ — (2)

Mesh 3 $7.5i_3 + 5(i_3 - i_2) = 0$

$-5i_2 + 12.5i_3 = 0$ — (3)

Solving eqn's (1) (2) & (3): $i_1 = 6.42A$; $i_2 = 1.42A$

$i_3 = 0.57A$

$\therefore I'' = 0.57A$ (↓)

Total current through $(R_1) = I' + I'' = 2.5 + 0.57A$

$$I = 3.07A$$

6.a) Draw the series RLC circuit and derive the expression for resonant frequency and bandwidth.

Series Resonance:

- **Resonance:** A circuit containing reactance is said to be in resonance **if the voltage across the circuit is in phase with the current through it.** At resonance, the circuit thus behaves as a pure resistor and the net reactance is zero.
- Consider the **series RLC circuit** shown in **Fig. a.**

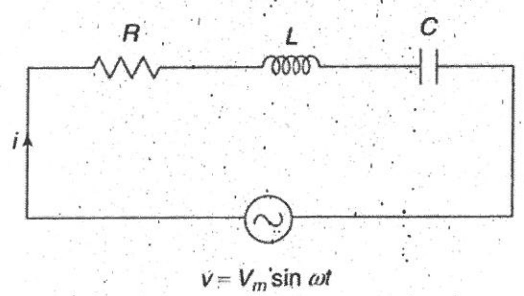


Fig. a *Series circuit*

$$Z = R + jX_L - jX_C = R + j(X_L - X_C)$$

At resonance, the circuit is purely resistive.

$$X_L - X_C = 0$$

$$X_L = X_C$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$\omega_0^2 = \frac{1}{LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

where f_0 is called the **resonant frequency** of the circuit.

Bandwidth:

- For the series RLC circuit, bandwidth is defined as **"the range of frequencies for which the power delivered to R is greater than or equal to $\frac{P_0}{2}$ where P_0 is the power delivered to R at resonance"**.
- From the shape of the resonance curve shown in **Fig.b**, it is clear that there are two frequencies for which the power delivered to R is half the power at resonance.
- For this reason, these frequencies are referred as those corresponding to the **half-power points.**
- The magnitude of the current at each half power point is the **same.**

$$P_1 = P_2 = \frac{P_0}{2} = \frac{I_0^2 R}{2}$$

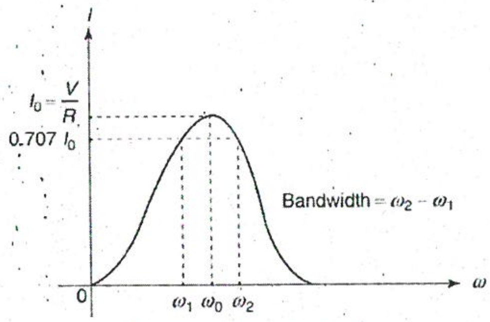


Fig. b *Resonance curve*

Hence,

$$I_1^2 R = \frac{1}{2} I_0^2 R = I_2^2 R$$

where the subscript 1 denotes the lower half point and the subscript 2, the higher half point.

It follows then that $I_1^2 = I_2^2 = \frac{1}{2}\frac{I_0^2 R}{R}$

$$I_1 = I_2 = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$

- The bandwidth may be identified on the resonance curve as the range of frequencies over which the **magnitude of the current is equal to or greater than 0.707 of the current** at resonance.
- In Fig. 6, the **bandwidth is $\omega_2 - \omega_1$.**

  ω_2 is called the **upper cut-off frequency**

  ω_1 is called the **lower cut-off frequency.**

Expression for Bandwidth:

Generally, at any frequency ω,

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \quad \text{...(i)}$$

At half-power points,

$$I = \frac{I_0}{\sqrt{2}}$$

But

$$I_0 = \frac{V}{R}$$

$$I = \frac{V}{\sqrt{2}R} \quad \text{...(ii)}$$

From Eqs (i) and (ii),

$$\frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{V}{\sqrt{2}R}$$

$$\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{2}R$$

Squaring both the sides,

$$R^2+\left(\omega L-\frac{1}{\omega C}\right)^2=2R^2$$

$$\left(\omega L-\frac{1}{\omega C}\right)^2=R^2$$

$$\omega L-\frac{1}{\omega C}\pm R=0$$

$$\omega^2\pm\frac{R}{L}\omega-\frac{1}{LC}=0$$

$$\omega=\pm\frac{R}{2L}\pm\sqrt{\frac{R^2}{4L^2}+\frac{1}{LC}}$$

For low values of R, the term $\left(\frac{R^2}{4L^2}\right)$ can be neglected in comparison with the term $\frac{1}{LC}$.

Then ω is given by, $\omega=\pm\frac{R}{2L}\pm\sqrt{\frac{1}{LC}}=\pm\frac{R}{2L}\pm\frac{1}{\sqrt{LC}}$

The resonant frequency for this circuit is given by

$$\omega_0=\frac{1}{\sqrt{LC}}$$

$$\omega=\pm\frac{R}{2L}+\omega_0$$ (considering only posititve sign of ω_0)

$$\omega_1=\omega_0-\frac{R}{2L}$$

and $$\omega_2=\omega_0+\frac{R}{2L}$$

$$f_1=f_0-\frac{R}{4\pi L}$$

and $$f_2=f_0+\frac{R}{4\pi L}$$

$$\text{Bandwidth}=\omega_2-\omega_1=\frac{R}{L}$$

or $$\text{Bandwidth}=f_2-f_1=\frac{R}{2\pi L}$$

6.b) Discuss about the quality factor of a series and parallel resonance circuit.

Quality Factor:

- It is a measure of selectivity or sharpness of the resonant circuit.

Series resonant circuit:

- It is measure of voltage magnification in the series resonant circuit.

$$Q_0 = \frac{\text{Voltage across inductor or capacitor}}{\text{Voltage at resonance}} = \frac{V_{L_0}}{V} = \frac{V_{C_0}}{V}$$

Substituting values of V_{L_0} and V,

$$Q_0 = \frac{I_0 X_{L_0}}{I_0 R} = \frac{X_{L_0}}{R} = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 CR}$$

Substituting values of ω_0,

$$Q_0 = \frac{\left(\frac{1}{\sqrt{LC}}\right)L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Parallel resonant circuit

- It is a measure of current magnification in a parallel resonant circuit.

$$Q_0 = \frac{\text{Current through inductor or capacitor}}{\text{Current at resonance}} = \frac{I_{C_0}}{I_0}$$

Substituting values of I_{C_0} and I_0;

$$Q_0 = \frac{\frac{V}{X_{C_0}}}{\frac{VCR}{L}} = \frac{\frac{1}{X_{C_0}}}{\frac{CR}{L}} = \frac{\omega_0 C}{\frac{CR}{L}} = \frac{\omega_0 L}{R}$$

Neglecting the resistance R, the resonant frequency ω_0 is given by

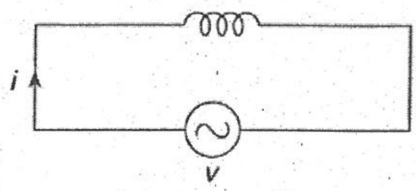
$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q_0 = \frac{\left(\frac{1}{\sqrt{LC}}\right)L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

7.a) What are coupled circuits and explain about self and mutual inductance.

Coupled circuits:

- Two circuits are said to be coupled circuits when **energy transfer takes place from one circuit to the other without having any electrical connection between them.**
- Such coupled circuits are frequently used in network analysis and synthesis.
- Common examples of coupled circuits are transformer, gyrator, etc.
- Consider a coil of N turns carrying a current i as shown in **Fig.**



Coil carrying current

- When current flows through the coil, a flux f is produced in the coil.

- The flux produced by the coil links with the coil itself.
- If the current flowing through the coil changes, the flux linking the coil also changes.
- Hence, an emf is induced in the coil. This is known as **self-induced emf.**
- The direction of this emf is given by Lenz's law

$$\phi \propto i$$
$$\frac{\phi}{i} = k, \text{ a constant}$$
$$\phi = k\, i$$

Hence, rate of change of flux = k × rate of change of current

$$\frac{d\phi}{dt} = k\frac{di}{dt}$$

According to Faraday's laws of electromagnetic induction, a self-induced emf can be expressed as

$$v = -N\frac{d\phi}{dt} = -Nk\frac{di}{dt} = -N\frac{\phi}{i}\frac{di}{dt} = -L\frac{di}{dt}$$

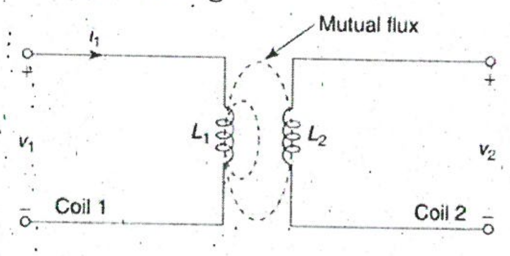
where $L = \frac{N\phi}{i}$ and is called coefficient of self-inductance.

We know that

- The property of a coil that opposes any change in the current flowing through it is called **self-inductance or inductance of the coil.**
- If the current in the coil is increasing, the self-induced emf is set up in such a direction so as to oppose the rise in current, i.e., the direction of self-induced emf is opposite to that of the applied voltage.
- Similarly, if the current in the coil is decreasing, the self-induced emf will be in the same direction as the applied voltage.
- Self-inductance **does not prevent the current from changing, it serves only to delay the change.**

Mutual Inductance:

- If the flux produced by one coil links with the other coil, placed closed to the first coil, an emf is induced in the second coil due to change in the flux produced by the first coil.
- This is known as **mutually induced emf.**
- Consider two coils 1 and 2 placed adjacent to each other as shown in **Fig.**
- Let Coil 1 has N_1 turns while Coil 2 has N_2 turns.
- If a current i_1 flows in Coil 1, flux is produced and a part of this flux links Coil 2.
- The emf induced in Coil 2 is **called mutually induced emf.**



Two adjacent coils

We know that

$$\phi_2 \propto i_1$$
$$\frac{\phi_2}{i_1} = k, \text{ a constant}$$
$$\phi_2 = k\, i_1$$

Hence, rate of change of flux = k × rate of change of current i_1.

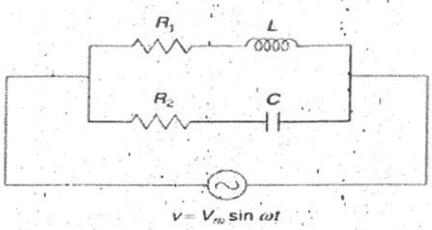
$$\frac{d\phi_2}{dt} = k\frac{di_1}{dt}$$

According to Faraday's law of electromagnetic induction, the induced emf is expressed as

$$v_2 = -N_2\frac{d\phi_2}{dt} = -N_2\, k\frac{di_1}{dt} = -N_2\frac{\phi_2}{i_1}\frac{di_1}{dt} = -M\frac{di_1}{dt}$$

where $M = \frac{N_2\phi_2}{i_1}$ and is called *coefficient of mutual inductance.*

7.b) Derive the expression for the resonant frequency of the given circuit.



$$Z_1 = R_1 + jX_L$$
$$Z_2 = R_2 - jX_C$$

For a parallel circuit;

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$Y = \frac{1}{R_1 + jX_L} + \frac{1}{R_2 - jX_C} = \frac{R_1 - jX_L}{R_1^2 + X_L^2} + \frac{R_2 + jX_C}{R_2^2 + X_C^2} = \frac{R_1}{R_1^2 + X_L^2} + \frac{R_2}{R_2^2 + X_C^2} - j\left(\frac{X_L}{R_1^2 + X_L^2} - \frac{X_C}{R_2^2 + X_C^2}\right)$$

At resonance, the circuit is purely resistive. Hence, the condition for resonance is

$$\frac{X_L}{R_1^2 + X_L^2} - \frac{X_C}{R_2^2 + X_C^2} = 0$$

$$\frac{X_L}{R_1^2 + X_L^2} = \frac{X_C}{R_2^2 + X_C^2}$$

$$\frac{\omega_0 L}{R_1^2 + \omega_0^2 L^2} = \frac{\frac{1}{\omega_0 C}}{R_2^2 + \frac{1}{\omega_0^2 C^2}}$$

$$\frac{\omega_0^2 LC}{R_1^2 + \omega_0^2 L^2} = \frac{\omega_0^2 C^2}{R_2^2 \omega_0^2 C^2 + 1}$$

$$LC(R_2^2 \omega_0^2 C^2 + 1) = C^2(R_1^2 + \omega_0^2 L^2)$$

$$\omega_0^2 R_2^2 LC^3 + LC = C^2 R_1^2 + \omega_0^2 L^2 C^2$$

$$\omega_0^2 R_2^2 LC^3 - \omega_0^2 L^2 C^2 = C^2 R_1^2 - LC$$

$$\omega_0^2 LC^2 (CR_2^2 - L) = C(CR_1^2 - L)$$

$$\omega_0^2 LC(CR_2^2 - L) = CR_1^2 - L$$

$$\omega_0^2 = \frac{CR_1^2 - L}{LC(CR_2^2 - L)}$$

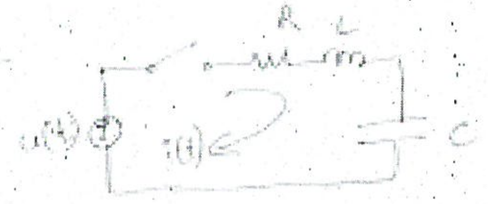
$$\omega_0 = \sqrt{\frac{CR_1^2 - L}{LC(CR_2^2 - L)}} = \frac{1}{\sqrt{LC}}\sqrt{\frac{CR_1^2 - L}{CR_2^2 - L}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{CR_1^2 - L}{CR_2^2 - L}}$$

where f_0 is called the resonant frequency of the circuit.

8.a) Derive the unit step current response of series RLC circuit.

8a



Transformed network

Applying KVL

$$\frac{1}{s} = \left(R + Ls + \frac{1}{Cs}\right) I(s)$$

$$I(s) = \frac{1}{s} \cdot \frac{1}{\left(R + Ls + \frac{1}{Cs}\right)} = \frac{1}{s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC}}$$

$$= \frac{1}{(s-s_1)(s-s_2)}$$

$$s_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$s_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

$$I(s) = \frac{1}{(s-s_1)(s-s_2)} = \frac{A}{s-s_1} + \frac{B}{s-s_2}$$

$$A = \frac{1}{s_1 - s_2} \quad \& \quad B = \frac{1}{s_2 - s_1}$$

$$\therefore I(s) = \frac{1}{s_1 - s_2}\left[\frac{1}{s-s_1} - \frac{1}{s-s_2}\right]$$

$$i(t) = \frac{1}{s_1 - s_2}\left[e^{s_1 t} - e^{s_2 t}\right] = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

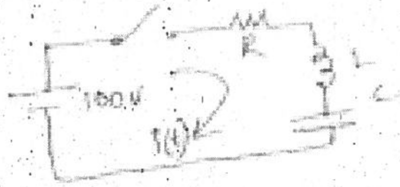
Depending on values of s_1 & s_2 the response is either overdamped, critically damped or under damped.

Note: consider differential equation solution also.

8.b) A series RLC circuit with R=50Ω, L=100 mH and C=50 μF as a voltage of 100V applied to it at t=0 through a switch. Evaluate the expression for a current transient. Assume initially relaxed circuit conditions.

8b

R = 50Ω ; L = 100mH ; C = 50μF ; V = 100V



Applying KVL

$V - Ri - L\frac{di}{dt} - \frac{1}{C}\int_0^t i\,dt = 0$

Differentiating the above equation

$0 - R\frac{di}{dt} - L\frac{d^2i}{dt^2} - \frac{1}{C}i = 0$

$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i = 0$

This is second order differential equation, the characteristic equation is

$s^2 + \left(\frac{R}{L}\right)s + \frac{1}{LC} = 0$

The roots are

$s_1 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

$s_2 = -\frac{R}{2L} - \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

Substituting R, L & C values

$s_1 = -250 + j370.8 = -\alpha + j\omega_d$

$s_2 = -250 - j370.8 = -\alpha - j\omega_d$

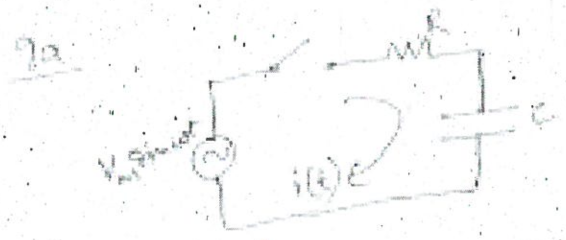
Solution is $i(t) = k_1 e^{s_1 t} + k_2 e^{s_2 t}$

$= e^{-\alpha t}\{(k_1 + k_2)\cos\omega_d t + j(k_1 - k_2)\sin\omega_d t\}$

$k_1 + k_2 = 0 \Rightarrow$ by substituting initial conditions

9.a) Derive an expression for the transient current in series RC circuit with a sinusoidal source using differential equations.

9a



$$v(t) = Ri(t) + \frac{1}{C}\int i\,dt = V_m \sin\omega t$$

differentiating on both sides

$$R\frac{di}{dt} + \frac{i}{C} = V_m \cos\omega t\ (\omega)$$

$$\left(\frac{di}{dt} + \frac{1}{RC}\right) i = \frac{V_m\omega}{R}\cos\omega t$$

This is of the form of non-homogeneous differential equation.

$$\frac{di}{dt} + Pi = Q$$

sol'n is $i(t) = e^{-Pt}\int Q e^{Pt}\,dt + K e^{-Pt}$

$$P = \frac{1}{RC} \quad ; \quad Q = \frac{V_m\omega}{R}\cos\omega t$$

$$i(t) = e^{-\frac{1}{RC}t}\int \frac{V_m\omega}{R}\cos\omega t \cdot e^{\frac{1}{RC}t}\,dt + K e^{-\frac{1}{RC}t}$$

$$= e^{-\frac{1}{RC}t}\int \frac{V_m\omega}{R}\left(\frac{e^{j\omega t} + e^{-j\omega t}}{2}\right) e^{\frac{1}{RC}t}\,dt + K e^{-\frac{1}{RC}t}$$

$$= \frac{V_m\omega RC}{2}\left[\frac{e^{j\omega t}}{1+j\omega RC} + \frac{e^{-j\omega t}}{1+j\omega RC}\right] + K e^{-\frac{1}{RC}t}$$

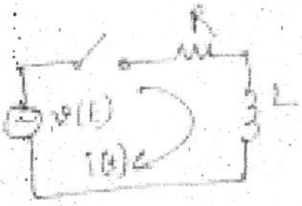
At $t=0 \Rightarrow i(t) = \dfrac{V_m \sin\omega t}{R} = 0$

$$K = \frac{-V_m\omega RC}{1+(\omega RC)^2}$$

9.b) A voltage pulse v(t)= u(t - 2) - u(t - 4) is applied to a series RL circuit with R = 5 ohms and L= 5henry. Obtain voltage expression across R and L. Where u(t) is unit step function.

9b $v(t) = u(t-2) - u(t-4)$

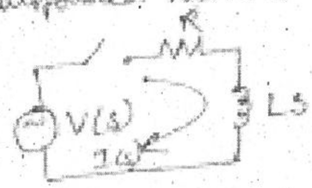
$R = 5\Omega \quad L = 5H$



$v(t) = u(t-2) - u(t-4)$

$V(s) = \frac{e^{-2s}}{s} - \frac{e^{-4s}}{s}$

Transformed network



$V(s) = (R + Ls) I(s)$

$\frac{e^{-2s} - e^{-4s}}{s} = (5 + 5s) I(s)$

$I(s) = \left(e^{-2s} - e^{-4s}\right) \frac{1}{5s(s+1)}$

Considering

$\frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$

$A = 1 \; ; \; B = -1$

$I(s) = \frac{1}{5} e^{-2s}\left(\frac{1}{s} - \frac{1}{s+1}\right) - \frac{1}{5} e^{-4s}\left(\frac{1}{s} - \frac{1}{s+1}\right)$

$i(t) = \frac{1}{5}\left[u(t-2) - e^{-(t-2)} u(t-2) - u(t-4) + e^{-(t-4)} u(t-4)\right]$

10.a) Derive the relationship between transmission (ABCD) parameters and open circuit impedance (Z) parameters.

Z-parameter in Terms of ABCD Parameters We know that

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

Rewriting the second equation,

$$V_2 = \frac{1}{C}I_1 + \frac{D}{C}I_2$$

Comparing with $V_2 = Z_{21}I_1 + Z_{22}I_2$,

$$Z_{21} = \frac{1}{C}$$
$$Z_{22} = \frac{D}{C}$$

Also,
$$V_1 = A\left[\frac{1}{C}I_1 + \frac{D}{C}I_2\right] - BI_2 = \frac{A}{C}I_1 + \left[\frac{AD}{C} - B\right]I_2 = \frac{A}{C}I_1 + \left[\frac{AD - BC}{C}\right]I_2$$

Comparing with $V_1 = Z_{11}I_1 + Z_{12}I_2$,

$$Z_{11} = \frac{A}{C}$$
$$Z_{12} = \frac{AD - BC}{C}$$

(OR)

ABCD Parameters in Terms of Z-parameters We know that

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Rewriting the second equation,

$$I_1 = \frac{1}{Z_{21}}V_2 - \frac{Z_{22}}{Z_{21}}I_2$$

Comparing with $I_1 = CV_2 - DI_2$,

$$C = \frac{1}{Z_{21}}$$
$$D = \frac{Z_{22}}{Z_{21}}$$

Also,
$$V_1 = Z_{11}\left[\frac{1}{Z_{21}}V_2 - \frac{Z_{22}}{Z_{21}}I_2\right] + Z_{12}I_2 = \frac{Z_{11}}{Z_{21}}V_2 - \frac{Z_{22}Z_{11}}{Z_{21}}I_2 + Z_{12}I_2$$
$$= \frac{Z_{11}}{Z_{21}}V_2 - \left[\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}\right]I_2$$

Comparing with $V_1 = AV_2 - BI_2$,

$$A = \frac{Z_{11}}{Z_{21}}$$
$$B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}} = \frac{\Delta Z}{Z_{21}}$$

10.b) Compute the transmission, parameters for the two-port network if the Z parameters for the network are Z_{11}= 42 Ω, Z_{22}= 35Ω, Z_{12}= Z_{21}=25Ω.

10 b

$Z_{11} = 42\Omega$; $Z_{22} = 35\Omega$; $Z_{12} = Z_{21} = 25\Omega$

$A = \frac{Z_{11}}{Z_{21}} = \frac{42}{25} = 1.68$

$B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}} = \frac{(42)(35)-(25)^2}{25} = 33.8$

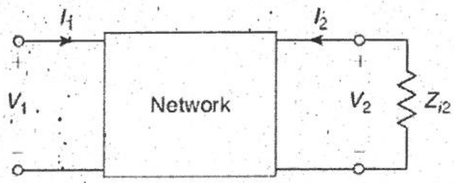
$C = \frac{1}{Z_{21}} = 0.04$

$D = \frac{Z_{22}}{Z_{21}} = \frac{35}{25} = 1.4$

11.a) Discuss about the image parameters for the symmetrical two port networks.

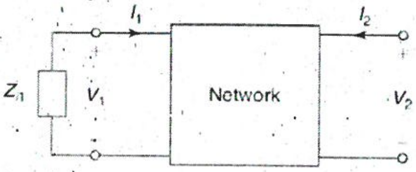
Image Parameters:

- If the driving-point impedance at port 1, with impedance Z_{i2} connected across port 2, is Z_{i1} and driving-point impedance at port 2, with impedance Z_{i1} connected across the port 1, is Z_{i2} then Z_{i1} and Z_{i2} are known as image impedances of the network.
- These are also known as image parameters.
- The image parameters can be expressed in terms of ABCD parameters. Figure shows a two-port network terminated in Z_{i2} at Port 2.



$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$
$$V_2 = -Z_{i2}I_2$$
$$Z_{i1} = \frac{V_1}{I_1} = \frac{AV_2 - BI_2}{CV_2 - DI_2} = \frac{-AZ_{i2}I_2 - BI_2}{-CZ_{i2}I_2 - DI_2} = \frac{-AZ_{i2} - B}{-CZ_{i2} - D}$$

$$= \frac{AZ_{i2} + B}{CZ_{i2} + D} \quad \ldots(13.13)$$



Similarly, if the two-port network is terminated in Z_{i1} at port 1 as shown in Fig. then

$$V_1 = -Z_{i1} I_1$$

$$Z_{i2} = \frac{V_2}{I_2} = \frac{V_1\left(\frac{D}{AD-BC}\right) - I_1\left(\frac{B}{AD-BC}\right)}{V_1\left(\frac{C}{AD-BC}\right) - I_1\left(\frac{A}{AD-BC}\right)} \quad \ldots\text{(from Cramer's rule)}$$

$$= \frac{-Z_{i1}I_1\left(\frac{D}{AD-BC}\right) - I_1\left(\frac{B}{AD-BC}\right)}{-Z_{i1}I_1\left(\frac{C}{AD-BC}\right) - I_1\left(\frac{A}{AD-BC}\right)} = \frac{-DZ_{i1} - B}{-CZ_{i1} - A}$$

$$= \frac{DZ_{i1} + B}{CZ_{i1} + A} \quad \ldots(13.14)$$

Solving Eqs (13.13) and (13.14),

$$Z_{i1} = \sqrt{\frac{AB}{CD}}$$

$$Z_{i2} = \sqrt{\frac{BD}{AC}}$$

- The image impedances Z_{i1} and Z_{i2} do not define a network completely.
- A third parameter **image transfer constant Φ** is also used to define the network.
- Φ can also be expressed in terms of ABCD parameters.
- Let the network be terminated in impedance Z_{i2} at Port 2.

$$V_2 = -Z_{i2} I_2$$

$$V_1 = AV_2 - BI_2 = AV_2 - B\left(\frac{V_2}{-Z_{i2}}\right) = \left(A + \frac{B}{Z_{i2}}\right)V_2$$

$$I_1 = CV_2 - DI_2 = -CZ_{i2}I_2 - DI_2 = -(CZ_{i2} + D)I_2$$

$$\frac{V_1}{V_2} = A + \frac{B}{Z_{i2}} = A + B\sqrt{\frac{AC}{BD}} = A + \frac{\sqrt{ABCD}}{D} = \frac{AD + \sqrt{ABCD}}{D}$$

Also

$$\frac{I_1}{I_2} = -CZ_{i2} + D = C\sqrt{\frac{BD}{AC}} + D = \frac{\sqrt{ABCD}}{A} + D = \frac{\sqrt{ABCD} + AD}{A} = \frac{AD + \sqrt{ABCD}}{A}$$

Hence,

$$\frac{V_1}{V_2}\frac{I_1}{I_2} = \frac{(AD + \sqrt{ABCD})^2}{AD} = (\sqrt{AD} + \sqrt{BC})^2$$

$$\sqrt{\frac{V_1}{V_2}\frac{I_1}{I_2}} = \sqrt{AD} + \sqrt{BC}$$

$$= \sqrt{AD} + \sqrt{AD - 1} \quad (\because AD - BC = 1)$$

11.b) Derive the expression for image transfer constant (Φ) in terms of transmission (ABCD) parameters.

Open and short-circuit impedance parameters can be expressed in terms of $ABCD$ parameters.

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

When the output port is open-circuited, i.e., $I_2 = 0$,

$$V_1 = AV_2$$
$$I_1 = CV_2$$

Hence, the impedance measured at input port with output port open-circuited is

$$Z_{oc_1} = \frac{V_1}{I_1} = \frac{A}{C}$$

When the output port is short-circuited, i.e., $V_2 = 0$,

$$V_1 = -BI_2$$
$$I_1 = -DI_2$$

Hence, the impedance measured at input port with output port short-circuited is

$$Z_{sc_1} = \frac{V_1}{I_1} = \frac{B}{D}$$

A Similarly the impedance measured at output port with input port open-circuited and short-circuited are

$$Z_{oc_2} = \frac{D}{C}$$

$$Z_{sc_2} = \frac{B}{A}$$

The ratio of short-circuit to open-circuit impedance at the two ports is

$$\frac{Z_{sc_1}}{Z_{oc_1}} = \frac{Z_{sc_2}}{Z_{oc_2}} = \frac{BC}{AD}$$

The image parameters can also be expressed in terms of open circuit and short-circuit impedances.

$$Z_{i1} = \sqrt{\frac{AB}{CD}} = \sqrt{Z_{oc_1} Z_{sc_1}}$$

$$Z_{i2} = \sqrt{\frac{BD}{AC}} = \sqrt{Z_{oc_2} Z_{sc_2}}$$

$$\phi = \tanh^{-1}\sqrt{\frac{BC}{AD}} = \tanh^{-1}\sqrt{\frac{Z_{sc_1}}{Z_{oc_1}}} = \tanh^{-1}\sqrt{\frac{Z_{sc_2}}{Z_{oc_2}}}$$