Code: 23BS1201

I B.Tech - II Semester – Regular / Supplementary Examinations
JUNE 2026

DIFFERENTIAL EQUATIONS & VECTOR CALCULUS

(Common for ALL BRANCHES)

Duration: 3 hours Max. Marks: 70

Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

BL – Blooms Level CO – Course Outcome

PART – A

		BL	CO
1.a)	Solve the first order linear differential equation $\frac{dy}{dx} - 2y = e^x$.	L3	CO2
1.b)	Find the integrating factor for $(1+xy)\, y\, dx + (1-xy)\, x\, dy = 0$.	L2	CO1
1.c)	Find the general solution of $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 0$.	L2	CO2
1.d)	Define Wronskian.	L1	CO1
1.e)	Form the partial differential equation by eliminating the arbitrary constants a, b from the equation $z = ax + by$.	L3	CO2
1.f)	Solve $4\frac{\partial^2 z}{\partial x^2} + 12\frac{\partial^2 z}{\partial x\, \partial y} + 9\frac{\partial^2 z}{\partial y^2} = 0$.	L3	CO2
1.g)	If $\bar{r} = x\,\bar{\imath} + y\,\bar{\jmath} + z\bar{k}$, show that $\nabla \times \bar{r} = \bar{0}$.	L3	CO3
1.h)	Compute div $\bar{F}$ for the vector $\bar{F} = x^2 y\bar{\imath} + (3x - yz)\bar{\jmath} + z^3\bar{k}$.	L2	CO3

1.i)	State Gauss divergence theorem.	L2	CO1
1.j)	Calculate the line integral $\int_C (4xz + 2y)\, dx$, where C is the line segment from $(2, 1, 0)$ to (4, 0, 2).	L3	CO5

PART – B

			BL	CO	Max. Marks
UNIT-I					
2	a)	Solve $(1 + 2\mathrm{xy}\cos \mathrm{x}^2 - 2\mathrm{xy})\, \mathrm{dx} + (\sin \mathrm{x}^2 - \mathrm{x}^2)\, \mathrm{dy} = 0$.	L3	CO2	5 M
	b)	A cup of coffee is $180°\mathrm{F}$ when freshly poured. After 2 minutes in a room at $70°\mathrm{F}$, the coffee cools to $165°\mathrm{F}$. Find the temperature at any time t and the time when the coffee reaches $120°\mathrm{F}$	L4	CO4	5 M
OR					
3	a)	If the rate of decay of a radioactive substance is proportional to the amount present and it reduces to half in 5 days, how much will remain after 15 days?	L4	CO4	5 M
	b)	Solve $\mathrm{x}\frac{\mathrm{dy}}{\mathrm{dx}} + \mathrm{y} = \mathrm{x}^3\mathrm{y}^6$	L3	CO2	5 M
UNIT-II					
4	a)	Solve the linear ordinary differential equation $\frac{d^2y}{dx^2} + 4\frac{\mathrm{dy}}{\mathrm{dx}} + 4y = e^{-2x}\sin 2x$	L4	CO2	5 M
	b)	Using method of variation of parameters, find the general solution of non-homogeneous differential equation $\frac{d^2y}{dx^2} + y = \sec x$	L4	CO4	5 M

OR					
5	For an L-C-R circuit with inductance $L = \frac{1}{5}$ henries, resistance $R = 1$ ohm, and capacitance $C = \frac{5}{6}$ farads, connected to a voltage source $E(t) = \sin t$. Using the governing equation $L\frac{d^2q}{dt^2} + R\frac{\mathrm{dq}}{\mathrm{dt}} + \frac{q}{C} = E(t)$, find the charge $q(t)$ and the current $\mathrm{i}(t)$ at any time t.		L3	CO2	10 M
UNIT-III					
6	a)	Solve $x(\mathrm{y}-\mathrm{z})\frac{\partial z}{\partial x} + \mathrm{y}(\mathrm{z}-\mathrm{x})\frac{\partial z}{\partial y} = \mathrm{z}(\mathrm{x}-\mathrm{y})$.	L3	CO2	5 M
	b)	Solve $\frac{\partial^2 z}{\partial x^2} - 3\frac{\partial^2 z}{\partial y^2} = x^2$.	L4	CO4	5 M
OR					
7	a)	Form the partial differential equation by eliminating the arbitrary constants a, b from $(\mathrm{x}-\mathrm{a})^2 + (\mathrm{y}-\mathrm{b})^2 + \mathrm{z}^2 = \mathrm{c}^2$.	L3	CO2	5 M
	b)	Solve $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x\,\partial y} - 6\frac{\partial^2 z}{\partial y^2} = e^{x+y}$	L4	CO4	5 M
UNIT-IV					
8	a)	Find the angle between the surfaces $\mathrm{x}^2 + \mathrm{y}^2 + \mathrm{z}^2 = 9$ and $\mathrm{z} = \mathrm{x}^2 + \mathrm{y}^2 - 3$ at the point $(2, -1, 2)$.	L3	CO1	5 M
	b)	Find the directional derivative of the function $\phi(x, y) = x^2y - 4y^3$ at $(2, 1)$ in the direction of the unit vector $\bar{u} = \frac{\sqrt{3}}{2}\,\bar{\imath} + \frac{1}{2}\bar{\jmath}$.	L3	CO2	5 M

OR					
9	a)	Is the magnetic field $\bar{F} = (2xy - 3)\,\bar{\imath} + (x^2 + \cos y)\,\bar{\jmath}$ irrotational or not? If it is, find a corresponding scalar potential function.	L4	CO3	5 M
	b)	Calculate curl $\bar{F}$ at the point (1, 2, 3) given $\bar{F} = 3x^2\bar{\imath} + 5xy^2\bar{\jmath} + 5xyz^3\bar{k}$.	L3	CO3	5 M
UNIT-V					
10	a)	Using Green's theorem, evaluate $\int_C (xy + y^2)\,dx + x^2\,dy$, where C is bounded by $y = x$ and $y = x^2$.	L4	CO5	5 M
	b)	Find the circulation of the field $\bar{F} = (x - y)\,\bar{\imath} + x\,\bar{\jmath}$ around the circle $r(t) = \cos t\,\bar{\imath} + \sin t\,\bar{\jmath}, 0 \le t \le 2\pi$.	L3	CO5	5 M
OR					
11	a)	Evaluate $\int\int_Q\int 6xy\,dz\,dy\,dx$, where Q is the tetrahedron bounded by the planes $0 \le x \le 2$, $0 \le y \le 4 - 2x$ and $0 \le z \le 4 - 2x - y$.	L4	CO5	5 M
	b)	Evaluate $\int (x^2 + y^2)\,dx - 2xy\,dy$, over the region bounded by the lines $x = \pm a$, $y = 0$, $y = b$ using Stoke's theorem.	L4	CO5	5 M

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Differential Equations & Vector Calculus (23BS1201)

Scheme of Valuation. PART-A

(1a) Given eq is a linear eq is of the form $\frac{dy}{dx}+P(x)y = Q(x)$

$I.F = e^{\int P(x)dx} = e^{-2x} \longrightarrow$ (1M)

General solution is $y.(I.F) = \int Q(x)(I.F)dx + c$

$y.e^{-2x} = \int e^{x}.(e^{-2x})dx + c$

$= -e^{-x} + c \longrightarrow$ (1M)

(1b)

Given eq is of the form $Mdx + Ndy = 0$.

$M = y(1+xy)$; $N = x(1-xy)$.

$\frac{\partial M}{\partial y} = 1+2xy$. $\frac{\partial N}{\partial x} = 1-2xy \longrightarrow$ (1M)

$I.F = \frac{1}{Mx-Ny} = \frac{1}{2x^2y^2} \longrightarrow$ (1M)

(1c) The given eq can be written in operator form.

$(D^2+6D+9)x = 0 \Rightarrow A.E$ is $m^2+6m+9=0$. $D = \frac{d}{dt}$

$\Rightarrow m = -3,-3 \longrightarrow$ (1M)

$\therefore x = (c_1+c_2t)e^{-3t}$; c_1, c_2 are constants $\longrightarrow$ (1M)

(1d). The Wronskian of two functions $y_1(x)$ and $y_2(x)$ denoted by $W(y_1,y_2)$ is defined as $W(y_1,y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} \longrightarrow$ (2M)

(1e) differentiate partially w.r.to x and y of given eq $z = ax+by$ — (1).

$\frac{\partial z}{\partial x} = p = a$; $\frac{\partial z}{\partial y} = q = b$. $\longrightarrow$ (1M)

put these values in (1)

$\therefore z = px+qy \longrightarrow$ (1M)

(1f) Given eq in symbolic form is $(4D^2+12DD'+9D'^2)z=0$.
replace D by m & D' by 1

A.E is $4m^2+12m+9=0 \Rightarrow (2m+3)^2=0$.

$\Rightarrow m=-3/2, -3/2 \longrightarrow$ (1m)

$\therefore z = f_1(y-\frac{3}{2}x) + x \cdot f_2(y-\frac{3}{2}x) \longrightarrow$ (1m)

(1g)

$$\nabla \times \bar{r} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x & y & z \end{vmatrix} \longrightarrow \text{(1m)}$$

$= \bar{i}(0-0) + \bar{j}(0-0) + \bar{k}(0-0) = \bar{0} \longleftarrow$ (1m)

(1h)

$\bar{F} = x^2y\bar{i} + (3x-yz)\bar{j} + z^3\bar{k}$

$\operatorname{div}.\bar{F} = \nabla \cdot F$

$= \left(\bar{i}\frac{\partial}{\partial x} + \bar{j}\frac{\partial}{\partial y} + \bar{k}\frac{\partial}{\partial z}\right)\cdot(x^2y\bar{i} + (3x-yz)\bar{j} + z^3\bar{k}) \longrightarrow$ (1

$= 2xy - z + 3z^2 \longrightarrow$ (1m)

(1i) Let S be a closed surface enclosing a volume V; if F is a continuously differentiable vector pt function, then
$\int_V \operatorname{div}.\bar{F}\, dv = \int_S \bar{F}.\bar{n}\, ds$. where $\bar{n}$ is the outward drawn normal vector at any pt of S $\longrightarrow$ (2m)

(1j) parametric form of the line

$\frac{x-2}{2} = \frac{y-1}{-1} = \frac{z-0}{2} = t$

$\Rightarrow x = 2+2t;\ y = 1-t;\ z = 2t \quad : 0 \le t \le 1 \longrightarrow$ (1m)

$\int_C (4xz+2y)\,dx = \int_{t=0}^{1} [4(2+2t)2t + 2(1-t)]\cdot 2dt$

$= 2\int_{t=0}^{1}(16t^2+14t+2)\,dt = 2\left(\frac{16}{3}+7+2\right) = \frac{86}{3} \longrightarrow$ (1m)

(2a) Given eq of the form $Mdx + Ndy = 0$.

$M = 1 + 2xy\cos x^2 - 2xy$; $N = \sin x^2 - x^2$

$\frac{\partial M}{\partial y} = 2x\cos x^2 - 2x$; $\frac{\partial N}{\partial x} = \cos x^2 \cdot 2x - 2x$.

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$: Given eq is exact eq. $\longrightarrow$ (2M)

Solution is $\int_{y\ \text{const}} M\,dx + \int (\text{terms of } N \text{ not containing } x)\,dy = c$ $\longrightarrow$ (1M)

$x + 2y\int x\cos x^2\,dx - 2y\int x\,dx = c$.

$x + 2y\left[\frac{1}{2}\sin x^2\right] - 2y \cdot \frac{x^2}{2} = c$

$x + y\sin x^2 - yx^2 = c$ $\longrightarrow$ (2M)

(2b) By Newton's law of cooling $\theta = \theta_0 + c e^{-kt}$ $\longrightarrow$ (1M)

$\theta = 180°F$ at $t = 0$. $\theta_0 = 70°F$; when $\theta = 120$: $t = ?$

$\theta = 165°F$ at $t = 2$

$180 = 70 + ce^{-k(0)}$

$\Rightarrow c = 110$.

$\therefore \theta = \theta_0 + ce^{-kt}$

$165 = 70 + 110e^{-2k}$

$\Rightarrow e^{-2k} = \frac{95}{110} \Rightarrow e^{-k} = \left(\frac{95}{110}\right)^{1/2}$ $\longrightarrow$ (1M)

$\therefore k = -0.07358$.

$\therefore \theta = 70 + 110e^{-0.07358t}$. $\longrightarrow$ (1M)

$120 = 70 + 110e^{-kt} \Rightarrow e^{-kt} = 5/11$

$\left(\frac{95}{110}\right)^{\frac{1}{2}t} = 5/11$

$\Rightarrow \frac{t}{2}\log\left(\frac{95}{110}\right) = \log(5/11) \Rightarrow \frac{t}{2} = \frac{\log 5 - \log 11}{\log 95 - \log 110}$ — (1M)

$1.9777 - 9.0413 = 5.3836$

$\Rightarrow t = 10.76.$ $\longrightarrow$ 1m

3a

$y = c e^{-kt}$. (law of natural decay) $\longrightarrow$ 1m

initially $y = y_0$ at $t = 0$.

let initially amount of substance be y_0

$y = \frac{y_0}{2}$ at $t = 5$ days.

$\therefore c = y_0.$

$\frac{y_0}{2} = y_0 e^{-5k} \Rightarrow k = \frac{\ln 2}{5} = -0.1386.$ $\longrightarrow$ 2m

at $t = 15$; $y_0 = ?$

$y = y_0 \cdot e^{-15k}$

$= y_0 (e^{-5k})(e^{-5k}) \cdot (e^{-5k})$

$= y_0 (1/2)(1/2)(1/2)$

$= \frac{1}{8} y_0.$

$\frac{1}{8}$ of initial amount remains. —— 2m

3b

Given eq. is Bernoulli's eq

$y^{-6} \frac{dy}{dx} + \frac{1}{y^5} \cdot \frac{1}{x} = x^2$ —— eq ①

put $u = y^{-5} \Rightarrow \frac{du}{dx} = -5 y^{-6} \frac{dy}{dx}.$

① $\Rightarrow -\frac{1}{5} \frac{du}{dx} + \frac{u}{x} = x^2$

$\Rightarrow \frac{du}{dx} - \frac{5u}{x} = -5x^2$ which is a linear eq in u. $\longrightarrow$ 2m

$I.F = e^{\int -5/x \, dx} = \frac{1}{x^5}$ $\longrightarrow$ 1m

Solution $u \cdot \frac{1}{x^5} = -5 \int x^{-3} dx \Rightarrow \frac{1}{x^5 y^5} = \frac{5}{2} \cdot \frac{1}{x^2} + c$ $\longrightarrow$ 2m

$[\because u = \frac{1}{y^5}]$

(4a) Given eq in operator form is $(D^2+4D+4)y = e^{-2x}\sin 2x$.

A.E is $m^2+4m+4=0 \Rightarrow m=-2,-2$

$$y_c = (c_1+c_2x)e^{-2x} \longrightarrow (2m)$$

$$y_p = \frac{1}{D^2+4D+4}\cdot e^{-2x}\sin 2x$$

$$= e^{-2x}\cdot\frac{1}{(D-2)^2+4(D-2)+4}\sin 2x$$

$$= e^{-2x}\cdot\frac{1}{D^2}\sin 2x$$

$$= e^{-2x}\cdot\frac{1}{-4}\sin 2x. \longrightarrow (2m)$$

$$y = y_c + y_p$$

$$= (c_1+c_2x)e^{-2x} - \frac{1}{4}e^{-2x}\sin 2x \longrightarrow (1m)$$

(4b) Given eq in operator form is $(D^2+1)y = \sec x$; AE is $m^2+1=0$

eq (1)

$m = \pm i$ $\therefore y_c = c_1\cos x + c_2\sin x$.

$$= c_1 u(x) + c_2 v(x). \longrightarrow (1m)$$

Where $u(x) = \cos x$; $v(x) = \sin x$.

let the particular integral of (1) be

$$y_p = A\cos x + B\sin x$$

$$= A\,u(x) + B\,v(x)$$

$$u\frac{dv}{dx} - v\frac{du}{dx} = 1.$$

By the method of variation of parameters

$$A = -\int \frac{vR}{u\frac{dv}{dx} - v\frac{du}{dx}}\,dx = -\int \frac{\sin x\cdot\sec x}{1}\,dx$$

$$= -\int \tan x\,dx$$

$$= -\log|\sec(x)|$$

$$B = \int \frac{u R}{u\frac{dv}{dx} - v\frac{du}{dx}}\,dx = \int \frac{\cos x}{\cos x}\,dx = \int 1\,dx = x.$$

$$\therefore y_p = A\cos x + B\sin x$$
$$= -\log|\sec x|\cdot\cos x + x\sin x \longrightarrow 1M$$

G.S is $y = y_c + y_p$

$$y = c_1\cos x + c_2\sin x - \log|\sec x|\cdot\cos x + x\sin x \longrightarrow 1M$$

5a) Given that $L = \frac{1}{5}H$; $R = 1\Omega$; $c = \frac{5}{6}F$; $E(t) = \sin t$

Governing eq is $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = E(t)$

$$\frac{1}{5}q'' + 1\cdot q' + \frac{6}{5}q = \sin t.$$

$$q'' + 5q' + 6q = 5\sin t. \qquad D = \frac{d}{dt}.$$

The above eq in operator form is $(D^2+5D+6)q = 5\sin t$

A.E is $m^2+5m+6=0 \Rightarrow m = -2, -3$

$$\therefore q_c = c_1e^{-2t} + c_2e^{-2t}. \longrightarrow 2M$$

$$q_p = \frac{1}{D^2+5D+6}5\sin t = \frac{5}{-1+5D+6}\sin t = \frac{5}{5}\frac{1}{D+1}\sin t$$

$$= \frac{D-1}{D^2-1}\sin t = \frac{-1}{2}(D-1)\sin t = -\frac{1}{2}[\cos t - \sin t]$$

$$= \frac{1}{2}\sin t - \frac{1}{2}\cos t \longrightarrow 2$$

$$\therefore q(t) = c_1e^{-2t} + c_2e^{-3t} + \frac{1}{2}\sin t - \frac{1}{2}\cos t. \longrightarrow 2M$$

Charge

$$i(t) = \frac{dq(t)}{dt} = \frac{d}{dt}\left[c_1e^{-2t} + c_2e^{-3t} + \frac{1}{2}\sin t - \frac{1}{2}\cos t\right] \longrightarrow 2M$$

current

$$= -2c_1e^{-2t} - 3c_2e^{-3t} + \frac{1}{2}\cos t + \frac{1}{2}\sin t \longrightarrow 2M$$

$\therefore$ given form: $pP + qQ = R$.

Lagranges subsidiary eqs are $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)} \xrightarrow{eq} (1) \longrightarrow (1)$$

let the Lagranges multipliers $l, m, n = 1, 1, 1$

$lP + mQ + nR = 0$.

$\Rightarrow l\,dx + m\,dy + n\,dz = 0$.

$1\,dx + 1\,dy + 1\,dz = 0$.

Integrating $x + y + z = c_1 \longrightarrow$ (2m)

Consider the 2nd set of multipliers: $l, m, n = \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$

$lP + mQ + nR = 0$.

$\Rightarrow l\,dx + m\,dy + n\,dz = 0$.

$\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz = 0$.

by integrating $\log x + \log y + \log z = \log c_2$

$\Rightarrow xyz = c_2$

$\therefore$ G.S is $\phi(x+y+z, xyz) = 0 \longrightarrow$ (2m)

(6b)

Given eq in operator form is $(D^2 - 3D'^2)z = x^2$.

A.E is $m^2 - 3 = 0 \Rightarrow m = \pm\sqrt{3}$.

$\therefore z_c = f_1(y + \sqrt{3}x) + f_2(y - \sqrt{3}x) \longrightarrow$ (2m)

$$z_p = \frac{1}{D^2 - 3D'^2} \cdot x^2 = \frac{1}{D^2\left[1 - \frac{3D'^2}{D^2}\right]} \cdot x^2 = \frac{1}{D^2}\left[1 - \frac{3D'^2}{D^2}\right]^{-1} \cdot x^2$$

$$= \frac{1}{D^2}\left[1 + \frac{3D'^2}{D^2} + \cdots\right]x^2 = \frac{1}{D^2}x^2 = \frac{x^4}{12} \longrightarrow \text{(2m)}$$

$\therefore$ G.S is $z = z_c + z_p$

$$= f_1(y + \sqrt{3}x) + f_2(y - \sqrt{3}x) + \frac{x^4}{12} \longrightarrow \text{(1m)}$$

(7a) Given p.d.e is $(x-a)^2+(y-b)^2+z^2=c^2$ — eq (1)

d.p. w.r.to x and y; $2(x-a)+2z\frac{\partial z}{\partial x}=0$.

$\Rightarrow x-a=-pz$ — (2)

$(y-b)=-qz$; put these values in eq (1) — (2)

$\therefore (1) \Rightarrow (p^2+q^2+1)z^2=c^2 \longrightarrow$ (1M)

(7b) The given eq in symbolic form is $(D^2+DD'-6D'^2)z=e^{x+y}$

A.E is $m^2+m-6=0 \Rightarrow m=2,-3$.

$\therefore z_c=f_1(y-3x)+f_2(y+2x)$. $\longrightarrow$ (2M)

$z_p=\frac{1}{D^2+DD'-6D'^2}\cdot e^{x+y}$. replace D by 1 and D' by 1

we get $z_p=\frac{1}{1+1-6}e^{x+y}=-\frac{1}{4}e^{x+y}$. $\longrightarrow$ (2M)

$\therefore z=z_c+z_p$

$=f_1(y-3x)+f_2(y+2x)-\frac{1}{4}e^{x+y}$ $\longrightarrow$ (1M)

(8a) let $\left.\begin{array}{l} f\equiv x^2+y^2+z^2-9=0 \\ g\equiv x^2+y^2-z-3=0 \end{array}\right\}$ be the given surfaces.

$\nabla f=2x\bar{i}+2y\bar{j}+2z\bar{k}$; $\nabla g=2x\bar{i}+2y\bar{j}-\bar{k}$

$\bar{n}_1=\nabla f|_{(2,-1,2)}=4\bar{i}-2\bar{j}+4\bar{k}$; $\nabla g|_{(2,-1,2)}=\bar{n}_2=4\bar{i}-2\bar{j}-\bar{k}$ $\longrightarrow$ (2M)

angle b/w two surfaces is angle b/w their normals at that pt.

$\cos\theta=\frac{\bar{n}_1\cdot\bar{n}_2}{|\bar{n}_1|\cdot|\bar{n}_2|}=\frac{(4\bar{i}-2\bar{j}+4\bar{k})\cdot(4\bar{i}-2\bar{j}-\bar{k})}{\sqrt{36}\cdot\sqrt{21}}$

$=\frac{16}{6\cdot\sqrt{21}}=\frac{8}{3\sqrt{21}}$ $\longrightarrow$ (2M)

$\therefore \theta=\cos^{-1}\left(\frac{8}{3\cdot\sqrt{21}}\right) \longrightarrow$ (1M)

$\nabla\phi = 2xy\,\bar{i} + (x^2 - 12y^2)\,\bar{j}$

$\nabla\phi\big|_{(2,1)} = 4\bar{i} - 8\bar{j}$; given that $\bar{u} = \frac{\sqrt{3}}{2}\bar{i} + \frac{1}{2}$

direction derivative is $\nabla\phi \cdot \frac{\bar{u}}{|\bar{u}|} \longrightarrow$ (1m)

$= \frac{(4\bar{i} - 8\bar{j}) \cdot \left(\frac{\sqrt{3}}{2}\bar{i} + \frac{1}{2}\bar{j}\right)}{3/4 + 1/4}$

$\left.\begin{array}{l} = \frac{4\cdot\sqrt{3}}{2} - 8/2 \\ = 2\cdot\sqrt{3} - 4 \end{array}\right\} \longrightarrow$ (2m)

(9a)

Given that $\bar{F} = (2xy - 3)\bar{i} + (x^2 + \cos y)\bar{j}$

$\text{curl}\,\bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xy-3 & x^2+\cos y & 0 \end{vmatrix}$

$= \bar{i}\,[0-0] - \bar{j}\,(0-0) + \bar{k}\,(2x - 2x)$

$= \bar{0}$ $\therefore$ $\bar{F}$ is irrotational vector $\longrightarrow$ (2m)

Scalar potential $\bar{F} = \nabla\phi$

we have $d\phi = \frac{\partial\phi}{\partial x}dx + \frac{\partial\phi}{\partial y}dy + \frac{\partial\phi}{\partial z}dz$

$= \nabla\phi \cdot d\bar{r}$

$= \bar{F}\cdot d\bar{r} \longrightarrow$ (1m)

$d\phi = \left[(2xy-3)\bar{i} + (x^2 + \cos y)\bar{j}\right]\cdot\left[dx\bar{i} + dy\bar{j} + dz\bar{k}\right]$

$= (2xy - 3)dx + (x^2 + \cos y)dy$.

integrating we get $\phi = \int(2xy-3)dx + \int(x^2 + \cos y)dy$

$= x^2y - 3x + \sin y + c \longrightarrow$ (2m)

(9b) $\text{curl}\,\bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x^2 & 5xy^2 & 5xyz^3 \end{vmatrix} \longrightarrow$ (2M)

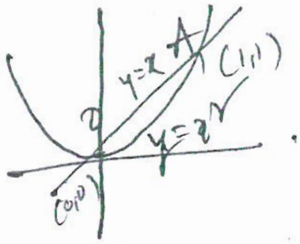
$= \bar{i}\left[\frac{\partial}{\partial y}(5xyz^3) - 0\right] - \bar{j}\left[\partial/\partial x(5xyz^3 - 0\right] + \bar{k}\left[\frac{\partial}{\partial x}(5xy^2 - 0)\right]$

$= \bar{i}\,[5xz^3] - \bar{j}\,[5yz^3] + \bar{k}\,[5y^2] \longrightarrow$ (2M)

$\text{curl}\,\bar{F}\,\Big|_{(1,2,3)} = 135\,\bar{i} - 270\,\bar{j} + 20\,\bar{k} \longrightarrow$ (1M)

(10a) By Green's th:- $\oint_C M\,dx + N\,dy = \iint \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy \longrightarrow$ (1M)

$M = xy + y^2;\ N = x^2 \Rightarrow \frac{\partial M}{\partial y} = x + 2y;\ \frac{\partial N}{\partial x} = 2x.$



$= \int_{x=0}^{1}\int_{y=x^2}^{x} (x - 2y)\,dy\,dx \longrightarrow$ (2M)

$= \int_{x=0}^{1} (x^2 - x^2) - (x^3 - x^4)\,dx$

$= \int_{x=0}^{1} (x^4 - x^3)\,dx = \frac{1}{5} - \frac{1}{4} = -\frac{1}{20} \longrightarrow$ (2M)

*

(or) along the line integrals also give full marks.

(10b) Given that $\bar{F} = (x-y)\,\bar{i} + x\,\bar{j}\ ;\ r(t) = \cos t\,\bar{i} + \sin t\,\bar{j}$

$d\bar{r} = -\sin t\,\bar{i} + \cos t\,\bar{j}$

circulation $= \int_C \bar{F}\cdot d\bar{r}$

$= (x-y)(-\sin t) + x\cos t \longrightarrow$ (2M)

put $x = \cos t;\ y = \sin t \Rightarrow dx = -\sin t\,dt;\ dy = \cos t\,dt.$

$\oint_C \bar{F}\cdot d\bar{r} = \int_{t=0}^{2\pi} \left[(\cos t - \sin t)(-\sin t) + \cos t\cdot\cos t\right] dt \longrightarrow$ (2M)

$= \int_{t=0}^{2\pi} \left(1 - \frac{\sin 2t}{2}\right) dt = \left(t - \frac{1}{4}\cos 2t\right)_{t=0}^{2\pi}$

$= 2\pi \longrightarrow$ (1M)

(11a) $\int_{x=0}^{2}\int_{y=0}^{4-2x}\int_{z=0}^{4-2x-y} 6xy\,dz\,dy\,dx$

$= \int_{x=0}^{2}\int_{y=0}^{4-2x} 6xy(4-2x-y)\,dy\,dx \longrightarrow$ (1m)

$= 6\int_{x=0}^{2} x\left[2y^2 - xy^2 - \frac{y^3}{3}\right]_{y=0}^{4-2x}$

$= 6\int_{x=0}^{2} x\left[2(4-2x)^2 - x(4-2x)^2 - \frac{(4-2x)^3}{3}\right]dx$

$= 6\int_{x=0}^{2} x\cdot(4-2x)^2\left[2-x-\frac{(4-2x)}{3}\right]dx$

$= \cancel{6}^{2}\int_{x=0}^{2} x\cdot(4-2x)^2\cdot\left(\frac{2-x}{\cancel{3}}\right)dx \longrightarrow$ (2m)

$= 2\int_{x=0}^{2}\left(32x+24x^3-48x^2-4x^4\right)dx$

$= 2\left[32\cdot\frac{x^2}{2}+24\cdot\frac{x^4}{4}-48\frac{x^3}{3}-4\frac{x^5}{5}\right]_{x=0}^{2}$

$= 2\left[160-\frac{128}{5}\right]$

$= \frac{64}{5} \longrightarrow$ (2m)

(11b). By Stoke's th. $\oint_C \bar{F}\cdot\bar{dr} = \int_S \text{curl}\,\bar{F}\cdot\hat{n}\,ds$.

$\int (x^2+y^2)dx - 2xy\,dy = \int \bar{F}\cdot\bar{dr}$

$\therefore \bar{F} = (x^2+y^2)\bar{i} - 2xy\bar{j}$

$$\text{curl}\,\bar{F} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2+y^2 & -2xy & 0 \end{vmatrix} = -4y\bar{k} \longrightarrow \text{(2m)}$$

let the surface is in xy-plane: $n = \bar{k}$

$ds = dx\,dy$.

$$\int_S \text{curl}\,\bar{F}\cdot n\,ds = \int_S -4y\,\bar{k}\cdot\bar{k}\,dx\,dy$$

$$= \int_{x=-a}^{a}\int_{y=0}^{b} -4y\,dx\,dy \longrightarrow \text{(2m)}$$

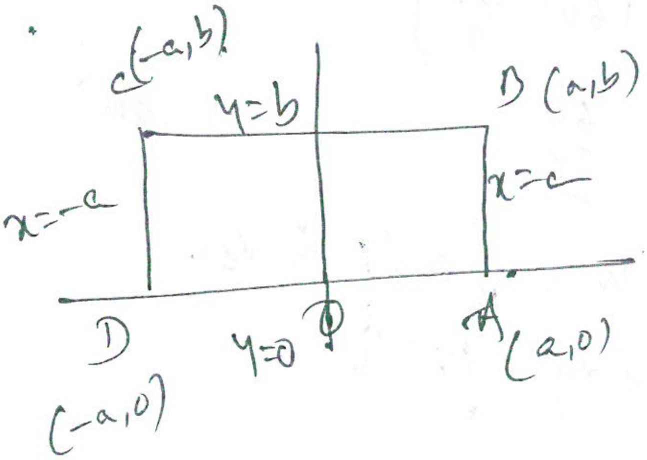
$$= -4\int_{y=0}^{b} y\,(x)_{x=-a}^{a}\,dy$$

$$= -4\int_{y=0}^{b} 2ay\,dy$$

$$= -4a\left(\cancel{2}y^2/\cancel{2}\right)_{y=0}^{b} \longrightarrow \text{(1m)}$$

$$= -4ab^2$$

(or)



$$\oint_C \bar{F}\cdot d\bar{r} = \int_{AD} + \int_{BC} + \int_{CD} + \int_{DA}$$

$$= -ab^2 - \frac{2a^3}{3} - 2ab^2 - ab^2 + \frac{2a^3}{3}$$

$$= -4ab^2$$