9		A balanced three phase three wire system has a Y-connected load. Each phase contains three loads in parallel: -j 100 Ω, 100 Ω and 50 + j50 Ω. Assume positive phase sequence with V_{ab} = 400volts. Find (i) V_{an} (ii) I_a A (iii) The power factor of the load (iv) The total power drawn by the load	L3	CO4	10 M
		UNIT-V			
10		Write a Short note on : i. Constant k Low pass filter ii. Constant k High pass filter	L2	CO5	10 M
		OR			
11	a)	Design a constant k low pass T Section filter having cut off frequency of 4khz and normal characteristic impedence of 500Ω.	L3	CO5	6 M
	b)	Calculate the cut-off frequency of active high pass filter circuit using a 330kΩ resistor and 100pF capacitor.	L3	CO5	4 M

Code: 23EE3301

PVP 23

II B.Tech - I Semester – Regular Examinations - DECEMBER 2024

ELECTRICAL CIRCUIT ANALYSIS - II
(ELECTRICAL & ELECTRONICS ENGINEERING)

Duration: 3 hours Max. Marks: 70

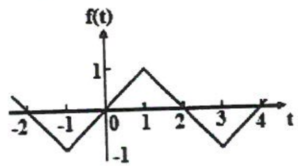
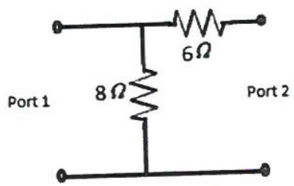
Note: 1. This question paper contains two Parts A and B.
2. Part-A contains 10 short answer questions. Each Question carries 2 Marks.
3. Part-B contains 5 essay questions with an internal choice from each unit. Each Question carries 10 marks.
4. All parts of Question paper must be answered in one place.

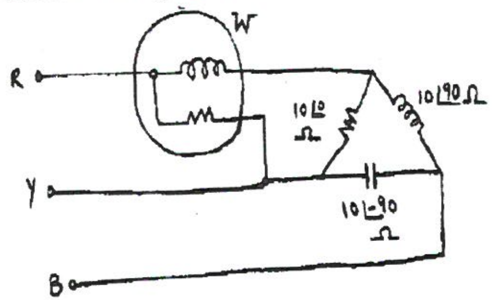
BL – Blooms Level CO – Course Outcome

PART – A

		BL	CO
1. a)	State any two applications of Laplace transform.	L2	CO1
b)	Mention the properties of Laplace transform.	L2	CO1
c)	Express the condition for reciprocity and symmetry in a two port Z - parameter representation.	L3	CO2
d)	What are called Admittance parameters?	L1	CO2
e)	Draw the transient growth and decay curves for an L –R circuit.	L2	CO3
f)	List the properties of RLC Series circuit.	L1	CO3
g)	Explain the difference between "balanced" and "unbalanced" load.	L2	CO4
h)	Why three phase systems are preferred over single-phase systems for the transmission of power?	L2	CO4
i)	Differentiate low pass and High pass filter.	L2	CO5
j)	What do you mean by Passive filter?	L2	CO5

PART – B

			BL	CO	Max. Marks
		UNIT-I			
2		Calculate the Fourier series for the function shown in figure 1.  Figure (1)	L3	CO1	10 M
		OR			
3	a)	Find the Laplace transform of $e^{-at}u(t)$.	L3	CO1	5 M
	b)	Determine the Inverse Laplace transform of $\frac{6(S+2)}{(S+1)(S+3)(S+4)}$	L3	CO1	5 M
		UNIT-II			
4		The Z-parameters of a two- port network is Z_{11}=15Ω, Z_{12}=Z_{21}=6Ω and Z_{22}=24Ω. Determine ABCD parameters.	L3	CO2	10 M
		OR			
5		For the network shown in the figure 2, determine ABCD parameters and using these parameters calculate impedance parameters.  Figure (2)	L3	CO2	10 M

			BL	CO	Max. Marks
		UNIT-III			
6		Derive the expression for the current in a series RL circuit (R = 10Ω, L = 10 mH) excited by a sinusoidal voltage of 100V, 50 Hz if the supply is connected at t = 0. Assume zero initial conditions.	L3	CO3	10 M
		OR			
7	a)	What is the condition for the response of a series RLC circuit excited by DC supply to have critically damped response?	L3	CO3	5 M
	b)	Draw the time response of inductor current in a series RL circuit excited by DC supply (step response).	L3	CO3	5 M
		UNIT-IV			
8		An unbalanced connected load is connected across a balanced 3 phase RYB 440V supply. Find the wattmeter reading connected in the circuit shown in figure 3.  figure (3)	L3	CO4	10 M
		OR			

II B.TECH I SEMESTER REGULAR EXAMINATION –DECEMBER 2024

PVP SIDDHARTHA INSTITUTE OF TECHNOLOGY

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

ELECTRICAL CIRCUIT ANALYSIS-II (23EE3301)

SCHEME OF EVALUATION

PART-A

Q.No	Description	Marks
1. a)	Any two applications	2M
b)	Any two properties	2M
c)	Condition for Symmetry	1M
	Condition for reciprocity	1M
d)	Admittance parameters definition	1M
	Y parameter equations	1M
e)	Transient Growth curve	1M
	Transient decay curve	1M
f)	Any two properties	2M
g)	Balanced Load definition	1M
	Unbalanced Load definition	1M
h)	Any two advantages of three phase over single phase system	2M
i)	Any two differences between low pass filter and high pass filter	2M
j)	Definition of passive filter	2M

PART-B

		Description	Marks
2		Determination of given triangular waveform in equation form	3M
		Fourier series equations	2M
		Evaluation of fourier coefficients	4M
		Representation of the fourier equation of the waveform	1M
3	a)	General form of laplace transform	2M
		Substitution of the function and determination of the laplace transform	3M
	b)	Determination of residues	3M
		Finding the inverse laplace of each term	2M
4		Representation of ABCD parameters in terms of Z parameters	5M
		Calculation of ABCD parameters	5M
5		Determination of ABCD parameters for the given circuit	5M

Conversion of Z parameters from ABCD parameters 5M

6 Determination of Impedance and phase angle 5M

Calculation of steady state current 3M

Expressing the final steady state equation 2M

7 a) Respresentation of series RLC circuit in standard second order differential equation 3M

Determination of roots of the 2^{nd} order ODE 1M

Determination of condition for critically damped system 1M

b) Obtaining the expression for current flowing through inductor excited by DC supply 4M

Plotting the current response of inductor current 1M

8 Determination of phase currents 3M

Line currents 3M

Power factor 2M

Expression for wattmeter reading and calculation 2M

9 Determination of phase impedance per phase 3M

Phase Voltage 2M

Line current 2M

Power factor of the load 1M

Total power drawn by the load 2M

10 a) Constant K Low pass filter 5M

- Nominal Impedance
- Cut off frequency
- Attenuation Constant
- Phase constant
- Characteristic impedance
- Design of filter

Any three parameter determination award marks

b) Constant K High pass filter 5M

- Nominal Impedance
- Cut off frequency
- Attenuation Constant
- Phase constant
- Characteristic impedance
- Design of filter

Any three parameter determination award marks

11 a) Design formula for constant low pass filter 2M

Determination of L and C 2M

Draw constant K low pass T section filter 2M

b) Cut-off frequency formula for constant high pass filter 2M

Calculation of Cut off frequency 2M

II B.TECH I SEMESTER REGULAR EXAMINATION –DECEMBER 2024

PVP SIDDHARTHA INSTITUTE OF TECHNOLOGY

DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING

ELECTRICAL CIRCUIT ANALYSIS-II (23EE3301)

A.Y 2024-25

1

a) The Laplace transform simplifies the analysis of linear time-invariant systems by converting time-domain differential equations into algebraic equations in the s-domain.
It helps determine transient behavior, and steady-state performance with transfer functions in the s-domain.

b) **Linearity**: $\mathcal{L}\{af(t)+bg(t)\}=a\mathcal{L}\{f(t)\}+b\mathcal{L}\{g(t)\}$

Time Shifting: $\mathcal{L}\{f(t-a)u(t-a)\}=e^{-as}\mathcal{L}\{f(t)\}$

Frequency Shifting: $\mathcal{L}\{e^{at}f(t)\}=F(s-a)$

Scaling in Time: $\mathcal{L}\{f(at)\}=\frac{1}{a}F\left(\frac{s}{a}\right)$

Differentiation in Time Domain: $\mathcal{L}\{f'(t)\}=sF(s)-f(0)$

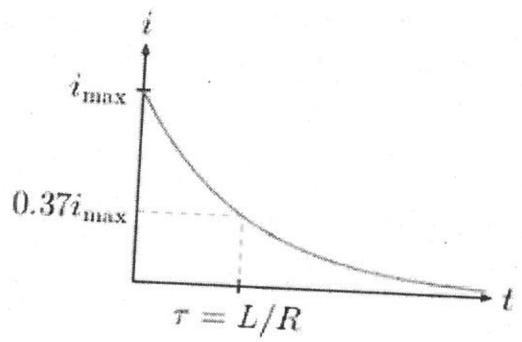
Integration in Time Domain: $\mathcal{L}\left\{\int_0^t f(\tau)d\tau\right\}=\frac{F(s)}{s}$

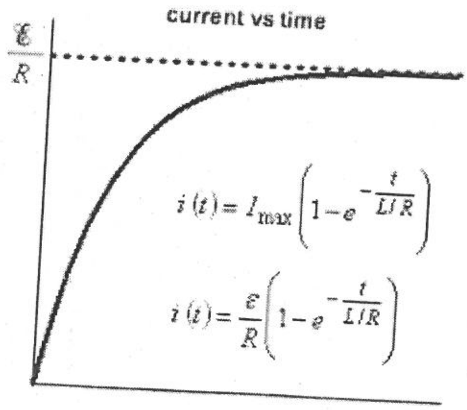
c) **Condition for Reciprocity**: $Z_{12} = Z_{21}$
Condition for Symmetry: $Z_{11} = Z_{22}$

d) **Admittance parameters (Y-parameters)** are a set of parameters used to describe the behavior of a two-port network in terms of input and output currents (I_1 and I_2) and voltages (V_1 and V_2).

$$I_1 = Y_{11}V_1 + Y_{12}V_2,$$
$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

e)





f) The **series RLC circuit** is a type of electrical circuit that contains a resistor (R), inductor (L), and capacitor (C) connected in series

Impedance: $Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$,

Resonance: $f_0 = \frac{1}{2\pi\sqrt{LC}}$.

Bandwidth: $\Delta f = \frac{R}{2\pi L}$.

g)

Balanced Load: A load is said to be balanced when the impedance in all three phases of the system is identical.

Unbalanced Load: A load is unbalanced when the impedance in the three phases is not identical.

Aspect	**Balanced Load**	**Unbalanced Load**
Impedance	Equal in all three phases.	Unequal in the three phases.
Phase Currents	Equal magnitude, 120° apart.	Unequal magnitudes and/or phases.
Voltage	Equal magnitude across all phases.	Unequal voltage drops across phases.

h) Three-phase systems are preferred over single-phase systems for power transmission due to following

1. **Higher Efficiency in Power Transmission**
2. Higher Power Capacity
3. Ease of Starting and Operation of Motors
4. cost-effective for long-distance transmission due to their higher efficiency, reduced and lower losses.

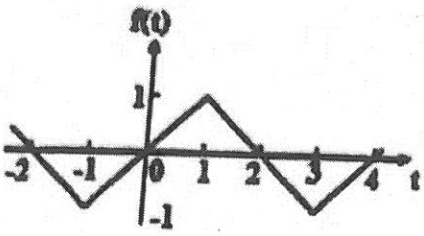
i)

Aspect	**Low-Pass Filter**	**High-Pass Filter**
Definition	Allows signals with frequencies below the cutoff frequency to pass through and attenuates higher frequencies.	Allows signals with frequencies above the cutoff frequency to pass through and attenuates lower frequencies.
Frequency Response	Passes low frequencies, attenuates high frequencies.	Passes high frequencies, attenuates low frequencies.
Circuit Type (Example)	- A resistor and capacitor in series, with output taken across the capacitor.	- A resistor and capacitor in series, with output taken across the resistor.

j) A passive filter is a type of electrical filter that uses only passive components, such as resistors (R), inductors (L), and capacitors (C), to filter signals by allowing certain frequency ranges to pass while attenuating others.

PART -B

2)



The function shown is a triangular periodic waveform with period T=4
This is an even function, so the sine terms in its Fourier series will vanish, and we only need to compute the cosine terms.
The general Fourier series for a periodic function f(t) with period T is:

$$f(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t)$$

where a_0 and a_n are given by:

$$a_0 = \frac{1}{T}\int_0^T f(t)dt, \quad a_n = \frac{2}{T}\int_0^T f(t)\cos(n\omega_0 t)dt$$

Since the function is symmetric about the time axis, its average value is zero. $a_0 = 0$

For a triangular wave, the Fourier series contains only odd harmonics (n=1,3,5,…), and the a_n coefficients decay as $1/n^2$

$$f(t) = \frac{8}{\pi^2} \sum_{n=1,3,5,\ldots}^{\infty} \frac{(-1)^{\frac{n-1}{2}}}{n^2} \cos(n\omega_0 t)$$

3)

a) definition of the Laplace transform:

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} f(t)e^{-st}\,dt.$$

Here, $f(t) = e^{at}$ Substituting f(t) into the formula, we get:

$$\mathcal{L}\{e^{at}\} = \int_0^\infty e^{at}e^{-st}\,dt.$$

$$\mathcal{L}\{e^{at}\} = \int_0^\infty e^{(a-s)t}\,dt.$$

Let α=a−s. The expression becomes

$$\mathcal{L}\{e^{at}\} = \int_0^\infty e^{\alpha t}\,dt.$$

The integral of $e^{\alpha t}$ is: $\int e^{\alpha t}\,dt = \frac{1}{\alpha}e^{\alpha t}$.

$$\mathcal{L}\{e^{at}\} = \left[\frac{1}{\alpha}e^{\alpha t}\right]_0^\infty.$$

Thus $\mathcal{L}\{e^{at}\} = \frac{1}{\alpha}(0-1) = -\frac{1}{\alpha}$.

Substituting $\alpha = a - s$ $\qquad \mathcal{L}\{e^{at}\} = -\frac{1}{a-s}$.

Rewriting we get $\mathcal{L}\{e^{at}\} = \frac{1}{s+a}$.

3 b)

To find the inverse Laplace transform, decompose $\frac{6(s+2)}{(s+1)(s+3)(s+4)}$ into partial fractions:

$$\frac{6(s+2)}{(s+1)(s+3)(s+4)} = \frac{A}{s+1} + \frac{B}{s+3} + \frac{C}{s+4}. \quad = \frac{1}{s+1} + \frac{3}{s+3} - \frac{4}{s+4}$$

Solving for coefficients A, B, and C (using substitution or equating coefficients):

A-2, B=-6 and C=10

$$e^{-t} + 3e^{-3t} - 4e^{-4t}$$

Therefore, $\frac{6(s+2)}{(s+1)(s+3)(s+4)} = \frac{2}{s+1} - \frac{6}{s+3} + \frac{10}{s+4}$.

Taking the inverse Laplace transform:

$$\mathcal{L}^{-1}\left\{\frac{2}{s+1} - \frac{6}{s+3} + \frac{10}{s+4}\right\} = 2e^{-t} - 6e^{-3t} + 10e^{-4t}.$$

$6(s+2) = A(s+3)(s+4) + B(s+1)(s+4) + C(s+1)(s+3)$

$-6 = B(-2)(1) \Rightarrow B = 3$

-3 $\quad -12 = C(-3)(-1) \Rightarrow C = -4$

-4 $\quad 6 = A(2)(3) \Rightarrow A = 1$

UNIT-II

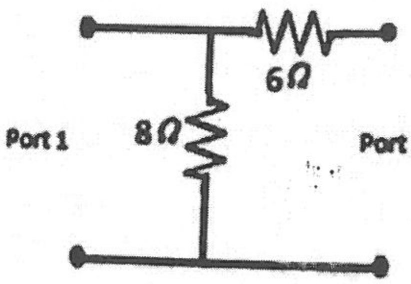
4. The Z-parameters are $Z_{11} = 15\Omega, Z_{12} = Z_{21} = 6\Omega, Z_{22} = 24\Omega$

The ABCD parameters can be calculated using

$$A = \frac{Z_{11}}{Z_{21}}, B = Z_{11} - \frac{Z_{12}Z_{21}}{Z_{22}}, C = \frac{1}{Z_{21}}, D = \frac{Z_{22}}{Z_{21}}.$$

Substituting and calculating we get A = 2.5, B=13.5Ω, C=0.1667 siemens and D=4

5.



The ABCD parameter equations are given by

$$A = \frac{V_1}{V_2}\Big|_{I_2=0}, \quad B = \frac{V_1}{I_2}\Big|_{V_2=0}, \quad C = \frac{I_1}{V_2}\Big|_{I_2=0}, \quad D = \frac{I_1}{I_2}\Big|_{V_2=0}.$$

Solving we get,

A = 1, B = 8+6 = 14Ω, C= 0 and D = 1

If C=0, the Z-parameters are undefined because the network does not allow current to flow in both ports simultaneously.

since C=0, this network represents a simple **series network**, and impedance parameters reduce to direct series addition:

Z_{11} = 14 Ω, Z_{12}=0, Z_{21}=0 and Z_{22}=14 Ω

6.

Given R=10Ω and L=0.01H

Voltage $v(t) = 100\sin(2\pi \cdot 50t)$ and the initial conditions are zero

The series RL circuit obeys the equation: $v(t) = Ri(t) + L\frac{di(t)}{dt}$.

Substituting $v(t) = 100\sin(2\pi \cdot 50t)$, we get

$$100\sin(2\pi \cdot 50t) = 10i(t) + 0.01\frac{di(t)}{dt}.$$

Rewriting equation, $\frac{di(t)}{dt} + \frac{R}{L}i(t) = \frac{v(t)}{L}.$

Here, $\frac{R}{L} = \frac{10}{0.01} = 1000$

The equation becomes $\frac{di(t)}{dt} + 1000i(t) = \frac{100\sin(2\pi \cdot 50t)}{0.01} = 10^4 \sin(100\pi t).$

Using the **particular solution** approach and solving, the steady-state current will be

$$i(t) = I_m \sin(2\pi \cdot 50t - \phi),$$

$|Z| = \sqrt{R^2 + (\omega L)^2} = \sqrt{10^2 + (\pi)^2}. = 10.48\Omega$

$I_m = \frac{V_m}{|Z|}. = \frac{100}{10.48} = 9.54\text{ A}$ $\qquad \phi = \tan^{-1}\left(\frac{\omega L}{R}\right). = 17.44^0$

The current in the circuit is $i(t) = 9.54\sin(314t - 17.44)$

7 a)

Condition for Critically Damped Response of RLC Circuit

For a series RLC circuit, the standard second-order differential equation is

$$L\frac{d^2i(t)}{dt^2} + R\frac{di(t)}{dt} + \frac{i(t)}{C} = 0$$

The damping condition is determined by the **damping factor** ζ, given as $\zeta = \frac{R}{2\sqrt{L/C}}.$

The response is **critically damped** when ζ=1. Thus $\frac{R}{2\sqrt{L/C}} = 1 \Rightarrow R = 2\sqrt{\frac{L}{C}}.$

Condition for critically damped response $R = 2\sqrt{\frac{L}{C}}.$

7 b) Time Response of Inductor Current in Series RL Circuit

Given a series RL circuit with DC step input V, the voltage equation is

$$V = Ri(t) + L\frac{di(t)}{dt}$$

$$\frac{di(t)}{dt} = \frac{V}{L} - \frac{R}{L}i(t)$$

Solving this first-order differential equation, the current response is

$$i(t) = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right).$$

The current starts at i(0)=0 and asymptotically approaches i(∞)=V/R. The time constant is τ=L/R.

8 The three-phase delta-connected load with the following details

Supply Voltage (Line-to-Line): V_{LL}=440 V

Z_{RY} = 10Ω (impedance between R and Y)

Z_{BR} = j10 Ω (impedance between B and R)

Z_{YB} = −j10 Ω (impedance between Y and B)

$$V_{ph} = \frac{V_{LL}}{\sqrt{3}} = \frac{440}{\sqrt{3}} \approx 254.03\text{ V}$$

In a delta configuration, the phase current I_{ph} is related to the phase voltage V_{ph} and the impedance Z

$$I_{ph} = \frac{V_{ph}}{Z}$$

For phase RY:

$$I_{RY} = \frac{V_{ph}}{Z_{RY}} = \frac{254.03}{10} \approx 25.40\text{ A}$$

For phase BR:

$$I_{BR} = \frac{V_{ph}}{Z_{BR}} = \frac{254.03}{j10} = \frac{254.03}{10} \cdot (-j) = -j \cdot 25.40\text{ A}$$

This is a purely imaginary current, indicating it is 90° out of phase with the voltage.

For phase YB:

$$I_{YB} = \frac{V_{ph}}{Z_{YB}} = \frac{254.03}{-j10} = \frac{254.03}{10} \cdot j = j \cdot 25.40\text{ A}$$

The line currents in a delta connection are related to the phase currents. The line current is the vector sum of two phase currents. For each phase

Line current in the R-phase:

$$I_R = I_{RY} - I_{BR} = 25.40\text{ A} - (-j25.40) = 25.40 + j25.40\text{ A}$$

Line current in the Y-phase:

$$I_Y = I_{BR} - I_{YB} = -j25.40 - j25.40 = -j50.80\text{ A}$$

Line current in the B-phase:

$$I_B = I_{YB} - I_{RY} = j25.40 - 25.40 = -25.40 + j25.40 \text{ A}$$

The magnitude of the line currents is given by

$$|I_R| = \sqrt{(25.40)^2 + (25.40)^2} \approx 35.94 \text{ A}$$

$$|I_Y| = 50.80 \text{ A}$$

$$|I_B| = \sqrt{(25.40)^2 + (25.40)^2} \approx 35.94 \text{ A}$$

The wattmeter is connected such that the current coil is in the R-phase and the potential coil is between R and Y. The power measured by the wattmeter is the active power given by

$$P = V_R \cdot I_R \cdot \cos(\theta)$$

$V_R = 254.03 \text{ V}$ (phase voltage between R and Y)

$I_R = 25.40 + j25.40 \text{ A}$ (line current in the R-phase)

The phase voltage V_R and the line current I_R are both complex numbers, so the power factor cos(θ) will depend on their phase difference.

Since the impedance between R and Y is real ($Z_{RY} = 10\,\Omega$), the phase difference between V_R and I_R will be zero (i.e., they are in phase). Therefore, the power factor $\cos(\theta) = 1$.

Thus, the power drawn by the wattmeter is:

$$P = 254.03 \text{ V} \times 25.40 \text{ A} \times 1 = 6451.76 \text{ W}$$

9 Each phase contains three parallel loads $Z_1 = -j100\ \Omega$, $Z_2 = 100\ \Omega$ and $Z_3 = 50 + j50\ \Omega$ Line voltage $V_{ab} = 400$ V and Y-connected load with positive phase sequence.

$$V_{\text{phase}} = V_{an} = \frac{V_{\text{line}}}{\sqrt{3}}$$

$$V_{an} = \frac{400}{\sqrt{3}} = 231\text{V(rms)}$$

The total impedance in each phase is the parallel combination of Z_1, Z_2, Z_3

$$\frac{1}{Z_{\text{total}}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$Y_1 = \frac{1}{Z_1} = \frac{1}{-j100} = j0.01\text{S},\ Y_2 = \frac{1}{Z_2} = \frac{1}{100} = 0.01\text{S and}$$

$$Y_3 = \frac{1}{Z_3} = \frac{1}{50 + j50} = \frac{50 - j50}{(50)^2 + (50)^2} = \frac{50 - j50}{5000} = 0.01 - j0.01\text{S}$$

$$Y_{\text{total}} = Y_1 + Y_2 + Y_3 = j0.01 + 0.01 + (0.01 - j0.01)$$

$$Z_{\text{total}} = \frac{1}{Y_{\text{total}}} = \frac{1}{0.03} = 33.33\Omega$$

$$I_a = \frac{V_{\text{phase}}}{Z_{\text{total}}} = \frac{231}{33.33} \approx 6.93\text{A}$$

The power factor is given by $\text{pf} = \cos\theta = \frac{\text{Real Part of } Z_{\text{total}}}{|Z_{\text{total}}|}$. Since the load is purely resistive.

$$\text{Power factor} = 1$$

The total power in a balanced 3-phase system is $P_{\text{total}} = 3V_{\text{phase}} I_{\text{phase}} \cos\theta$.

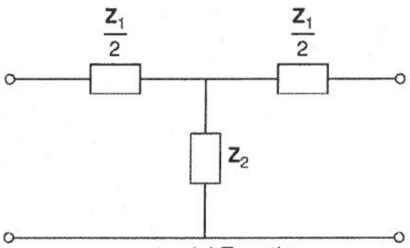
$$P_{\text{total}} = 3*231*6.93*1 = 4800\text{W}$$

10 Constant K Low Pass filter

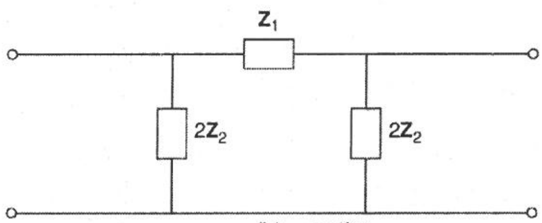
A T or π network is said to be of the constant k type if $\mathbf{Z}_1$ and $\mathbf{Z}_2$ are opposite types of reactances satisfying the relation

$$\mathbf{Z}_1\mathbf{Z}_2 = k^2$$

where k is a constant, independent of frequency. k is often referred to as *design impedance* or *nominal impedance* of the constant-k filter. The constant-k, T or π-type filter is also known as the prototype filter because other complex networks can be derived from it. Figure 15.4 shows constant-k, T and π-section filters.



(a) T-section



(b) π-section

In constant-k low pass filter,

$$\mathbf{Z}_1 = j\omega L$$

$$\mathbf{Z}_2 = -j\frac{1}{\omega c} = \frac{1}{j\omega c}$$

1. Nominal Impedance

$$k = \sqrt{\mathbf{Z}_1\mathbf{Z}_2} = \sqrt{(j\omega L)\left(\frac{1}{j\omega c}\right)} = \sqrt{\frac{L}{C}}$$

2. Cut-off Frequency

The cut-off frequencies are obtained when $\frac{\mathbf{Z}_1}{4\mathbf{Z}_2} = 0$ and $\frac{\mathbf{Z}_1}{4\mathbf{Z}_2} = -1$

(i) When

$$\frac{\mathbf{Z}_1}{4\mathbf{Z}_2} = 0$$

$$\mathbf{Z}_1 = 0$$

$$j\omega L = 0$$

$$\omega = 0$$

$$f = 0$$

(ii) When

$$\frac{\mathbf{Z}_1}{4\mathbf{Z}_2} = -1$$

$$\mathbf{Z}_1 = -4\mathbf{Z}_2$$

$$j\omega L = \frac{4j}{\omega C}$$

$$\omega^2 LC = 4$$

$$\omega^2 = \frac{4}{LC}$$

$$\omega = \omega_c = \frac{2}{\sqrt{LC}}$$

$$f = f_c = \frac{1}{\pi\sqrt{LC}}$$

Hence, the pass band starts at $f = 0$ and continues up to the cut-off frequency f_c. All the frequencies above f_c are in the attenuation or stop band.

3. Attenuation Constant

In pass band, $\alpha = 0$

In stop band, $\alpha = 2\cosh^{-1}\sqrt{\left|\frac{\mathbf{Z}_1}{4\mathbf{Z}_2}\right|} = 2\cosh^{-1}\sqrt{\frac{\omega^2 LC}{4}} = 2\cosh^{-1}\sqrt{\frac{\omega^2}{\omega_c^2}}$

$$= 2\cosh^{-1}\left(\frac{\omega}{\omega_c}\right) = 2\cosh^{-1}\left(\frac{f}{f_c}\right)$$

The attenuation constant α is zero throughout the pass band but increases gradually from the cut-off frequency.

4. Phase Constant

In pass band,

$$\beta = 2\sin^{-1}\sqrt{\left|\frac{\mathbf{Z}_1}{4\mathbf{Z}_2}\right|} = 2\sin^{-1}\left(\frac{f}{f_c}\right)$$

In stop band, $\beta = \pi$.

The phase constant β is zero at zero frequency and increases gradually through the pass band, reaches π at cut-off frequency f_c and remains at π for all frequency beyond f_c. The variation of β is plotted in Fig. 15.6.

5. Characteristic Impedance

$$\mathbf{Z}_{0T} = \sqrt{\frac{\mathbf{Z}_1^2}{4} + \mathbf{Z}_1\mathbf{Z}_2} = \sqrt{\mathbf{Z}_1\mathbf{Z}_2\left(1 + \frac{\mathbf{Z}_1}{4\mathbf{Z}_2}\right)}$$

$$= \sqrt{\frac{L}{C}\left(1 - \frac{\omega^2 LC}{4}\right)} = k\sqrt{1 - \left(\frac{\omega}{\omega_c}\right)^2} = k\sqrt{1 - \left(\frac{f}{f_c}\right)^2}$$

$$\mathbf{Z}_{0\pi} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_{0T}} = \frac{k}{\sqrt{1 - \left(\frac{f}{f_c}\right)^2}}$$

6. Design of Filter

$$k = \sqrt{\frac{L}{C}}$$

$$f_c = \frac{1}{\pi\sqrt{LC}}$$

Solving these two equations,

$$L = \frac{k}{\pi f_c}$$

$$C = \frac{1}{\pi f_c k}$$

Constant K High Pass filter

A constant-k high-pass filter is obtained by changing the positions of series and shunt arm of the constant-k low-pass filter, Figure 15.12 shows a constant-k, T and π section, high-pass filter.

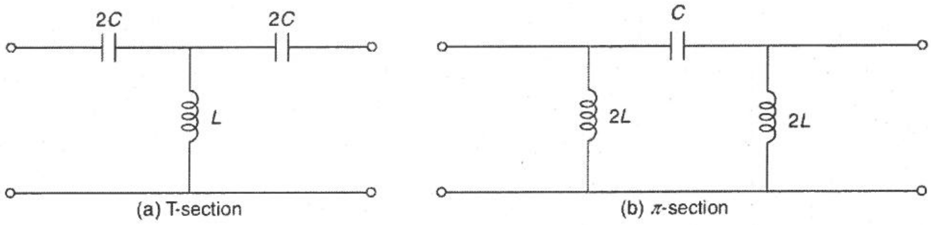


Fig. 15.12 *Constant-k high pass filter*

In a constant-k high-pass filter

$$\mathbf{Z}_1 = -j\frac{1}{\omega C} = \frac{1}{j\omega C}$$

$$\mathbf{Z}_2 = j\omega L$$

1. Nominal Impedance

$$k = \sqrt{\mathbf{Z}_1\mathbf{Z}_2} = \sqrt{\left(\frac{1}{j\omega C}\right)(j\omega L)} = \sqrt{\frac{L}{C}}$$

2. Cut-off Frequency

The cut-off frequencies are obtained when $\frac{\mathbf{Z}_1}{4\mathbf{Z}_2} = 0$ and $\frac{\mathbf{Z}_1}{4\mathbf{Z}_2} = -1$

(i) When

$$\frac{\mathbf{Z}_1}{4\mathbf{Z}_2} = 0$$

$$\mathbf{Z}_1 = 0$$

$$\frac{1}{j\omega C} = 0$$

$$\omega = \infty$$

$$f = \infty$$

(ii) When

$$\frac{\mathbf{Z}_1}{4\mathbf{Z}_2} = -1$$

$$\mathbf{Z}_1 = -4\mathbf{Z}_2$$

$$-j\frac{1}{\omega_c} = 4j\omega L$$

$$\omega^2 LC = \frac{1}{4}$$

$$\omega^2 = \frac{1}{4LC}$$

$$\omega = \omega_c = \frac{1}{2\sqrt{LC}}$$

$$f = f_c = \frac{1}{4\pi\sqrt{LC}}$$

Hence, the filter passes all the frequencies beyond f_c. The pass band starts at $f = f_c$ and continues up to infinite frequency. All the frequencies below the cut-off frequency lie in the attenuation or stop band.

3. Attenuation Constant

In pass band, $\alpha = 0$

In stop band, $\alpha = 2\cosh^{-1}\sqrt{\left|\frac{\mathbf{Z}_1}{4\mathbf{Z}_2}\right|} = 2\cosh^{-1}\left(\frac{f_c}{f}\right)$

The attenuation constant α decreases gradually to zero at the cut-off frequency and remains at zero through the pass band. The variation of α is plotted in Fig. 15.13.

4. Phase Constant

In pass band, $\beta = 2\sin^{-1}\sqrt{\left|\frac{\mathbf{Z}_1}{4\mathbf{Z}_2}\right|} = 2\sin^{-1}\left(\frac{f_c}{f}\right)$

In stop band, $\beta = \pi$.

The phase constant β remains at constant value π in the stop band and decreases to $-\pi$ in at f_c and reaches zero value gradually as f increases in the pass band. The variation of β is plotted in Fig. 15.14.

5. Characteristic Impedance

$$\mathbf{Z}_{0T} = \sqrt{\frac{\mathbf{Z}_1^2}{4} + \mathbf{Z}_1\mathbf{Z}_2} = \sqrt{\mathbf{Z}_1\mathbf{Z}_2\left(1 + \frac{\mathbf{Z}_1}{4\mathbf{Z}_2}\right)}$$

$$= \sqrt{\frac{L}{C}\left(1 - \frac{1}{4\omega^2 LC}\right)} = k\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

$$\mathbf{Z}_{0\pi} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_{0T}} = \frac{k}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

6. Design of Filter

$$k = \sqrt{\frac{L}{C}}$$

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

Solving these two equations,

$$L = \frac{k}{4\pi f_c}$$

$$C = \frac{1}{4\pi f_c k}$$

11 a)

Given:

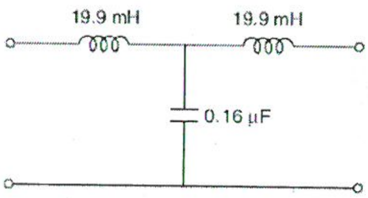
- Cut-off frequency f_c=4 kHz,
- Normal characteristic impedance Z_0=500 Ω.

Inductor (L) and Capacitor (C) Values: The design formulas are $L = \frac{Z_0}{\pi f_c}$, $C = \frac{1}{\pi Z_0 f_c}$

Substitute Z_0=500 Ω, f_c=4000Hz

$$L = \frac{500}{\pi \cdot 4000} \approx 0.0398\text{H} = 39.8\text{mH}$$

$$C = \frac{1}{\pi \cdot 500 \cdot 4000} \approx 159\text{nF}$$



11 b)

Given:

- Resistor R=330 kΩ,
- Capacitor C=100pF

The cut-off frequency for a high-pass filter is $f_c = \frac{1}{2\pi RC}$

$$f_c = \frac{1}{2\pi(330\times10^3)(100\times10^{-12})} = 4.83\text{kHz}$$